Corrigendum and addendum to
“Centralizers of finite subgroups in Hall’s universal group”

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Abstract – In Hall’s universal group every non-trivial conjugacy class satisfies \( CC = U \).
Hence generalized version of J. G. Thompson’s conjecture is true for every non-trivial
conjugacy class \( C \) in \( U \). Moreover Ore’s conjecture (every element is a commutator) is
true for \( U \) is added to [4]. In [4, Theorem 2.4] \( C_U(F)/Z(F) \cong U \) is true if \( Z(F) = 1 \).

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Let \( A \) be a periodic abelian group. In [1, p.10] and [2], a group \( G \) is called
a universal locally finite central extension of \( A \) if the following conditions are
satisfied.

(i) \( A \leq Z(G) \).
(ii) \( G \) is locally finite.

(iii) \( A \)-injectivity. Suppose that \( A \leq B \leq D \) with \( A \leq Z(D) \), that \( D/A \) is
finite, and that \( \psi : B \to G \) is an \( A \)-monomorphism (that is \( \psi(a) = a \) for all
\( a \in A \)). Then there exists an extension \( \tilde{\psi} : D \to G \) of \( \psi \) to a monomorphism
of \( D \) into \( G \). The class of all groups satisfying the above three conditions is
denoted by ULF(\( A \)). Hall’s universal group \( U \in ULF(1) \).

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Let $F$ be a finite subgroup of $U$. The group $U$ is an existentially closed group in the class of locally finite groups. Using this property, Hickin and Macintyre [3, Theorem 5] proved that $C_U(F)/Z(F)$ is a simple group. Our proof also shows that $C_U(F)$ is an extension of $Z(F)$. Hence if $F$ is finite abelian, then by [2, p. 53] $C_U(F) \in \text{ULF}(F)$.

**Remark 1.** If $F$ is finite and $Z(F) = 1$, then our proof shows that $C_U(F)$ is isomorphic to $U$. In the general case if $F$ is a finite subgroup of $U$ with non-trivial center, then $C_U(F)/Z(F)$ is not necessarily isomorphic to $U$. But quotient $C_U(F)/Z(F)$ is simple and it is a subgroup of $C_U(Z(F))/Z(F)$ where $C_U(Z(F)) \in \text{ULF}(Z(F))$. In particular in [4, Corollary 2.5], $C_U(F)$ has an epimorphic image isomorphic to $U$, should be replaced by $C_U(F)$ has a subgroup isomorphic to $U$.

In [5, Theorem 4.2] we use the same technique as in [4]. Therefore we notice that $C_G(F)/Z(F)$ is not necessarily isomorphic to $G$ for a subgroup $F$ contained in $G_i$ for some $i \in I$, unless $Z(F) = 1$. But by [5, Lemma 3.8] $C_G(F)/Z(F)$ is simple.

**Addendum**

Since in Hall’s universal group $U$ every finite subgroup $F$ is contained in a finite subgroup $B$ with $Z(B) = 1$, we have $U \cong C_U(B) \leq C_U(F)$. Then $U \cong C_U(B)Z(F)/Z(F) \leq C_U(F)/Z(F)$.

**Corollary 2.** The centralizer $C_U(F)$ of every finite subgroup $F$ of $U$ contains an isomorphic copy of $U$. Moreover $C_U(F)/Z(F)$ has a subgroup isomorphic to $U$.

**Corollary 3.** $U$ can be written as a direct limit of finite simple groups $G_1 \leq G_2 \leq G_i \leq \cdots$ where $U = \bigcup_{i \in \mathbb{N}} G_i$. Then $U$ has a descending chain of centralizers $C_U(G_i)$ where $C_U(G_1) \geq C_U(G_2) \geq C_U(G_3) \geq \cdots \geq C_U(G_i) \geq \cdots$ and for each $i \in \mathbb{N}$, $C_U(G_i) \cong U$ and $\bigcap_{i \in \mathbb{N}} C_U(G_i) = 1$

The property that $U$ is existentially closed in the class $LF$ of locally finite groups implies that every group $E$, existentially closed in any class $\mathcal{C}$ of groups satisfying $\mathcal{C} \supseteq LF$ will contain isomorphic copies of $U$.

One of the properties of $U$ is that, for every non-trivial conjugacy class $C$ in $U$ we have $C^2 = U$. It follows, clearly from this property that Generalized version of Thompson’s conjecture [6, p. @ 1069-2] for $U$ is true for any non-trivial conjugacy
class $C$ of $U$. The classification of finite simple groups is not used in the proof. Then the Ore conjecture: every element of $U$ is a commutator, follows immediately from Thompson’s conjecture.

By using free product, every infinite group $A$ generated by fewer than $\kappa$-elements can be embedded into a group $B$ generated by fewer than $\kappa$-elements with $Z(B) = 1$. Then we may repeat the above arguments for $U$, to $\kappa$-existentially closed groups and state the following.

**Corollary 4.** Let $G$ be the unique $\kappa$-existentially closed group of inaccessible cardinality $\kappa$ and $F$ be any proper subgroup of $G$. Then $C_G(F)$ contains a subgroup isomorphic to $G$. In particular if $Z(F) = 1$, then $C_G(F)$ is isomorphic to $G$. Moreover $C_G(F)/Z(F)$ has a subgroup isomorphic to $G$.

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**References**


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