On the Existence of Connecting Orbits

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Abstract. Two existence criteria of orbits connecting a pair of critical points of planar differential equations are given.

Keywords: Connecting orbits, parabolic sectors, exit points

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1. Introduction

In this paper, we consider the differential system

\[
\begin{align*}
\frac{dx}{dt} &= X(x, y) \\
\frac{dy}{dt} &= Y(x, y)
\end{align*}
\]

(1.1)

in the plane \( \mathbb{R}^2 \) where \( X \) and \( Y \) are continuous and assume that solutions of arbitrary initial value problems are unique. Let the vector field \( V = (X, Y) \) define a flow \( f(p, t) \) and let \( p_1, p_2 \in \mathbb{R}^2 \) be two isolated critical points of the system (1.1), i.e. \( V(p_1) = V(p_2) = 0 \).

Definition 1.1. If there is a point \( p_0 \in \mathbb{R}^2 \) such that

\[
\lim_{t \to +\infty} f(p_0, t) = p_1 \quad \text{and} \quad \lim_{t \to -\infty} f(p_0, t) = p_2,
\]

then \( f(p_0, \mathbb{R}) \) is called a trajectory connecting \( p_1 \) and \( p_2 \).

In some previous papers (see, e.g., [3, 4, 6]), generally it is assumed that one of two critical points \( p_1 \) and \( p_2 \) is a repeller or an attractor (about their definitions, see [3]). In the present paper we shall give some existence criteria for connecting orbits which contain no such assumption.

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2. Definitions

Let \( p \) be a simple closed curve surrounding a critical point \( Q \) of the system (1.1). Then, a positive or negative parabolic sector of \( Q \) in \( p \) is defined as open subset \( D \) of the interior of \( p \) with boundary consisting of

(i) the critical point \( Q \)
(ii) the positive or negative semi-trajectory arcs \( f(M_1, \mathbb{R}^+) \) and \( f(M_2, \mathbb{R}^+) \) or \( f(M_1, \mathbb{R}^-) \) and \( f(M_2, \mathbb{R}^-) \), respectively, and
(iii) the oriented closed subarc \( \rho_{12} \) from \( M_1 \) to \( M_2 \)
and such that when \( t \to +\infty \) or \( t \to -\infty \), then \( f(M_i, t) \to Q \) \((i = 1, 2)\) and the closure of \( D \) contains no negative or positive semi-trajectory \( f(M, \mathbb{R}^-) \) or \( f(M, \mathbb{R}^+) \) which tends to \( Q \) as \( t \to -\infty \) or \( t \to +\infty \), respectively, where \( M \in \rho_{12} \) (see [5: p. 163]).

**Definition 2.1.** A positive or negative parabolic sector \( D \) is said to be regular if the trajectory \( \Gamma(M) \) through any point \( M \in \rho_{12} \) is not tangent to \( \rho_{12} \) at \( M \) and \( f(M, t) \to Q \) as \( t \to +\infty \) or \( t \to -\infty \), respectively.

**Definition 2.2.** Let \( D \) be a regular positive or negative parabolic sector. An endpoint \( N_1 \in f(M_1, \mathbb{R}) \) of a simple curve \( \gamma \) is called an interior-side point of \( \gamma \) with respect to \( D \) if \( \gamma \) is not tangent to the trajectory \( f(M_1, \mathbb{R}) \) at \( N_1 \) and if there is a neighbourhood \( u(N_1) \) of \( N_1 \) on the curve \( \gamma \) such that either \( u(N_1) \subset D \) or the positive or negative semi-trajectory originating from any point in \( u(N_1) \) must intersect the closed subarc \( \rho_{12} \).

For \( N_2 \in f(M_2, \mathbb{R}) \), an interior-side point of \( \gamma \) with respect to \( D \) can be defined similarly.

Let \( T \subset \mathbb{R}^2 \) be an open subset, \( \overline{T} \) and \( \partial T \) its closure and boundary, respectively, and let \( f(p, [a, b]) \) denote the finite arc of the trajectory \( f(p, \mathbb{R}) \) corresponding to the interval \([a, b] \).

**Definition 2.3** (see [5: p. 37]). A point \( p_0 \in \partial T \) is called an exit point of \( T \) with respect to the system (1.1) if there exists an \( \varepsilon > 0 \) such that \( f(p_0, (\varepsilon, 0)) \subset T \). An exit point \( p_0 \) is called strict if there exists an \( \varepsilon > 0 \) such that \( f(p_0, (0, \varepsilon)) \subset \mathbb{R}^2 \setminus \overline{T} \).

An entrance point (a strict entrance point) can be defined similarly.

In what follows, the set of exit points of \( T \) will be denoted by \( S_1 \) and the set of strict exit points by \( S_1^* \).

Let

\[
T_0 = \left\{ p \in T \mid f(p, t_1) \not\subset T \text{ for some } t_1 > 0 \right\}
\]

and \( T_1 = T_0 \cup S_1 \). We define the function \( t_p : T_1 \to \mathbb{R} \) by

\[
t_p = \sup \left\{ t \geq 0 \mid f(p, [0, t]) \subset T \right\}.
\]

It is easy to see that \( f(p, [0, t_p]) \subset \overline{T} \) and \( f(p, t_p) \in S_1 \).

The following lemma holds (see also [2: p. 25]).
Lemma 2.1. Let $T \subset \mathbb{R}^2$ be an open subset satisfying $S_1 = S_1^*$, i.e. all exit points of $T$ are strict. Then the function $t_p$ defined by (2.2) is continuous.

Proof. By the condition $S_1 = S_1^*$ it follows that, for any given $p \in T$ and $\varepsilon > 0$,

$$f(p, [t_p, t_p + \varepsilon]) \not\subset T \cup S_1$$

and there is a point $t' \in (t_p, t_p + \varepsilon)$ such that $f(p, t') \not\in T$. Let $V$ be a neighbourhood of $f(p, t')$ in $\mathbb{R}^2$ which is disjoint from $\overline{T}$, and let $U$ be a neighbourhood of $p$ in $\mathbb{R}^2$ such that $f(U, t') \subset V$ (by the continuity of $f(p, t)$). Then, for $p' \in U \cap T$, $f(p', t') \not\in \overline{T}$. This implies $t_{p'} < t'$. Further, by $t' \in (t_p, t_p + \varepsilon)$ it follows that $t' \leq t_p + \varepsilon$. Therefore, $t_{p'} < t_p + \varepsilon$. This shows that $t_p$ is upper semicontinuous.

Now let $p \in T$ and let $\varepsilon > 0$ be arbitrarily given. By the definition of $t_p$, it follows that $f(p, [0, t_p - \varepsilon]) \subset T$, hence $f(p, [0, t_p - \varepsilon]) \cap \partial T = \emptyset$. Therefore, for every $\tau \in [0, t_p - \varepsilon]$, there is a neighbourhood $U_\tau$ of $f(p, \tau)$ in $\mathbb{R}^2$ which is disjoint from $\partial T$. Since the trajectory arc $f(p, [0, t_p - \varepsilon])$ is compact, a finite number of the $U_\tau$ cover $f(p, [0, t_p - \varepsilon])$. Let $\hat{U}$ be their union. Since $\hat{U}$ is open, there is a neighbourhood $\hat{V}$ of $p$ in $\mathbb{R}^2$ such that $f(\hat{V}, [0, t_p - \varepsilon]) \subset \hat{U}$. Since $\hat{U} \cap \partial T = \emptyset$, the inclusion $p' \in \hat{V}$ implies $f(p', [0, t_p - \varepsilon]) \cap \partial T = \emptyset$. Thus $t_{p'} > t_p - \varepsilon$, and $t_p$ is lower semicontinuous. This completes the proof.

Definition 2.4. If a simple closed curve $C$ is the union of alternating non-closed whole trajectories and critical points, and if it is contained in the $\omega$-limit set (or $\alpha$-limit set) of some trajectory, then we say that $C$ is a singular closed trajectory.

3. Results

In this section, we shall prove first the following theorem (see Figure 1).

Theorem 3.1. Suppose the following:

1. Let $p_1$ and $p_2$ be two critical points of the system (1.1) and let one of them, say $p_1$, has a regular negative parabolic sector $D_1$ in some simple closed curve $\rho_1$.

2. Let $M_1$ and $M_2$ be two endpoints of the oriented closed subarc $\rho_{12}$ of $D_1$.

3. Let $N_1, N_2$ be a simple curve connecting the points $N_i \in f(M_1, \mathbb{R}^+)$ and $N_i \in f(M_2, \mathbb{R}^+)$ such that each $N_i$ ($i = 1, 2$) is an interior-side point of $N_1N_2$ with respect to $D_1$.

4. Let $B$ be the region enclosed by the segmental arcs $M_1M_2, N_1N_2$ and the trajectory arcs $\overline{M_1N_1}, \overline{M_2N_2}$, and let the following three conditions be satisfied:

   (i) There is only one critical point $p_2$ in $B$.

   (ii) There are no closed trajectories and singular closed trajectories in $B$.

   (iii) All exit points of $B$ lying on the curve $N_1N_2$ are strict.

Then there must be in $B \cup \overline{D_1}$ a trajectory connecting $p_1$ and $p_2$ (see Figure 1).
**Proof.** Since $D_1$ is a regular negative parabolic sector, it follows from Definition 2.1 that every point of the segmental arc $M_1M_2$ is a strict entrance point of $B$. The theorem proof proceeds by reduction to absurdity. Suppose there are no trajectories joining $p_1$ and $p_2$. Then, by the Poincaré-Bendixson theory of planar systems and by the conditions (i) and (ii) it follows that the positive semi-trajectory $f(p, \mathbb{R}^+)$ originating from any point $p \in \overline{M_1M_2}$ must leave $B$ from the point $p' \in \overline{N_1N_2}$ for increasing time. It is easy to see that $p' \in S_1$ (where $S_1$ denotes the set of exit points of $B$). According to (2.2) we can define the function $t_p = \sup\{t \geq 0 | f(p, [0, t]) \subset B\}$ in the subset of $B$. Then $p' = f(p, t_p) \in \overline{N_1N_2}$ for $p \in \overline{M_1M_2}$. Let

$$K = \{ p' \in \overline{N_1N_2} | p' = f(p, t_p) \text{ for some } p \in \overline{M_1M_2} \}.$$ 

Now we can prove that $K = \overline{N_1N_2}$, i.e. every point of the arc $\overline{N_1N_2}$ is in $K$. In fact, by the conditions of the theorem, each of the $N_i$ ($i = 1, 2$) is an interior-side point of $\overline{N_1N_2}$ with respect to $D_1$. This means (see Definition 2.2) that every point in some neighbourhood of $N_i$ ($i = 1, 2$) on the arc $\overline{N_1N_2}$ is in $K$. Therefore, if there is a point $p_0' \in \overline{N_1N_2}$ such that $p_0'$ is not in $K$, then $K$ is a disconnected set on the arc $\overline{N_1N_2}$.

![Figure 1](image)

Since by Lemma 2.1 and by the continuity of the flow $f(p, t)$ it follows that $p' = f(p, t_p)$ is a continuous function of $p$, the arc $\overline{M_1M_2}$ will be mapped into a continuous segmental arc in $K$. But this is impossible because $K$ is disconnected and the images of $M_1$ and $M_2$ lie in two distinct connection components of $K$. Thus we have proved that $K = \overline{N_1N_2}$, hence $\overline{N_1N_2} \subset S_1$. Further, by the definition of $K$, we know that for every point $p_1' \in \overline{N_1N_2}$ there must be a point $p_1 \in \overline{M_1M_2}$ such that $p_1' = f(p_1, t_{p_1})$.

Consider now the mapping $p' = f(p, t_p)$ from $\overline{M_1M_2}$ to $\overline{N_1N_2}$. As stated above, it is surjective (onto). Further, if $f(p_1, t_{p_1}) = f(p_2, t_{p_2})$, then by the uniqueness of solutions it follows that $p_1 = p_2$. Hence this mapping is injective. Let

$$t_{p'} = \inf \{ t \leq 0 | f(p', [t, 0]) \subset B \}.$$ 

It is easy to see that $t_{p'}$ is defined for every point $p' \in \overline{N_1N_2}$. Thus we get the inverse map $p = f(p', t_{p'})$. Using the same argument used in Lemma 2.1 we can prove that the
inverse map is continuous. Hence \( p' = f(p, t_p) \) is a homeomorphism from \( M_1M_2 \) onto \( N_1N_2 \). Therefore \( M_1M_2 \) is mapped topologically onto \( N_1N_2 \) by trajectories of (1.1), and \( B \) is filled by these trajectories. But this contradicts the fact that \( p_2 \in B \). Hence Theorem 3.1 is proved.

**Remark.** In the case that there is a regular positive parabolic sector \( D \), a theorem similar to Theorem 3.1 can be proved provided we make the change \( t \to -t \) in the system (1.1).

**Theorem 3.2.** Suppose the following:

1. Let \( p_1 \) and \( p_2 \) be two critical points of the system (1.1) and let one of them, say \( p_1 \), have a positive parabolic sector \( D_1 \) with respect to some simple closed curve \( \rho_1 \).

2. Let \( M_1 \) and \( M_2 \) be two endpoints of the oriented closed subarc \( \rho_{12} \) of \( D_1 \), and let \( f(M_i, t) (i = 1, 2) \) are unbounded for \( t \to -\infty \).

3. Let \( B \) be an unbounded sectorial region bounded by two unbounded curves \( p_1M_i \cup f(M_i, \mathbb{R}^-) (i = 1, 2) \) with the same endpoint \( p_1 \) and containing \( D_1 \) in its interior, and let the following three conditions be satisfied in \( B \):
   
   (i) There is only one critical point \( p_2 \).
   
   (ii) There are no closed trajectories and singular closed trajectories.
   
   (iii) Every positive semi-trajectory of the system (1.1) is bounded.

Then there must be in \( B \) a trajectory connecting \( p_1 \) and \( p_2 \) (see Figure 2).

**Proof.** Consider the critical point \( p_2 \) and construct a circle \( \rho_2 \) of radius \( r \) with the centre \( p_2 \) such that \( \rho_2 \cap \rho_{12} = \emptyset \). We distinguish the three cases

(I) there is at least one hyperbolic sector of \( p_2 \) in \( \rho_2 \)

(II) there are no hyperbolic sectors of \( p_2 \), but it has at least an elliptic one in \( \rho_2 \)

(III) there are no hyperbolic and elliptic sectors of \( p_2 \) in \( \rho_2 \)

and consider them step by step.

Case (I): It is easy to see that there must be a point \( N_1 \in \rho_2 \) such that \( f(N_1, t) \to p_2 \) as \( t \to -\infty \). By condition (iii), the positive semi-trajectory \( f(N_1, \mathbb{R}^+) \) is bounded. Therefore, by the Poincaré-Bendixson theory of planar systems and condition (ii) we have \( f(N_1, t) \to p_1 \) as \( t \to +\infty \). Hence Theorem 3.2 holds.
Case (II): By [5: p. 164], there is at most a finite number of elliptic sectors of $p_2$ in $P_2$. Since there are no hyperbolic sectors, the number of elliptic sectors is even and there must be at least one negative parabolic sector. Thus there must be a point $N \in p_2$ such that the trajectory $f(N, \mathbb{R})$ connects $p_1$ and $p_2$. Hence Theorem 3.2 holds.

Case (III): In this case, all sectors of $p_2$ in $P_2$ are parabolic. If there is one base solution (see [5: p. 162]) $\gamma(t)$ such that $\gamma(t) \to p_2$ as $t \to -\infty$, then the same argument used in case (I) implies $\gamma(t) \to p_1$ as $t \to +\infty$. Hence Theorem 3.2 holds.

Therefore, in what follows, assume that all base solutions are positive, i.e. they tend to $p_2$ as $t \to +\infty$. Consider a positive semi-trajectory $f(q, \mathbb{R}^+)$ which tends to $p_2$ as $t \to +\infty$, where $q \in p_2$. It is easy to see that $f(q, t)$ must be unbounded for $t \to -\infty$ because otherwise its $\alpha$-limit set must contain critical points or closed orbits. But this contradicts conditions (i) and (ii). Therefore, there is a simple curve $MN$ connecting the point $M \in f(q, \mathbb{R}^-)$ and the point $N \in f(M_1, \mathbb{R}^-)$ or $N \in f(M_2, \mathbb{R}^-)$ such that $MN \cap p_2 = \emptyset$ and $MN \cap D_1 = \emptyset$ (see Figure 2). By [5: p. 169], after deleting the hyperbolic part and the elliptic portion in every parabolic sector, one obtains a subregion $S$ of the interior of $p_2$ such that the positive semi-trajectory through any point of $S$ is interior to $S$ for $t > 0$, and it tends to $p_2$ as $t \to +\infty$. Similarly, after deleting the hyperbolic part and the elliptic portion from $D_1$, one obtains a subregion $D'_1 \subset D_1$ and $D'_1$ possesses properties similar to $S$.

From the continuous dependence of solutions on initial conditions it follows that the positive semi-trajectory originating from any point in a small neighbourhood of $M$ on the curve $MN$ must enter $S$ for increasing time, thus it tends to $p_2$ as $t \to +\infty$, while the positive semi-trajectory originating from any point in a small neighbourhood of $N$ on the curve $MN$ must enter $D'_1$ for increasing time, thus it tends to $p_1$ as $t \to +\infty$. Similarly, for any point $p \in MN$, if $f(p, t) \to p_2$ or $f(p, t) \to p_1$ as $t \to +\infty$, there is an open neighbourhood $\sigma(p)$ of $p$ on the curve $MN$ such that for any point $\tilde{p} \in \sigma(p)$ the positive semi-trajectory $f(\tilde{p}, \mathbb{R}^+)$ must enter $S$ or $D'_1$ for increasing time, thus $f(\tilde{p}, t) \to p_2$ or $f(\tilde{p}, t) \to p_1$, respectively, as $t \to +\infty$. Therefore, there must be a point $Q \in MN$ such that $f(Q, t)$ tends to neither $p_2$ nor $p_1$ as $t \to +\infty$. By condition (iii), it follows that the $\omega$-limit set of $f(Q, t)$ contains some closed trajectory or some critical point different from $p_1$ and $p_2$. But this contradicts the conditions (i) and (ii).

4. An example

Consider the differential system

\[
\begin{align*}
\frac{dx}{dt} &= -\alpha \beta x^2 + \alpha xy \\
\frac{dy}{dt} &= y + \alpha y^2 - \alpha^2 x^2
\end{align*}
\]

in the plane $\mathbb{R}^2$ (see [1: p. 366]) where $\alpha > 0$ and $\beta > 0$ are constants. By [1: p. 367] we know that the critical point $O = (0, 0)$ of the system (4.1) is a saddle node, i.e. a critical point whose canonical neighbourhood is the union of one negative parabolic
sector and two hyperbolic sectors. The negative parabolic sector lies in the half-plane \( x < 0 \), its both boundary trajectories lie on the \( y \)-axis and all other trajectories of this sector tend to the origin \( O \) along the direction \( \theta = \pi \) as \( t \to -\infty \). The two hyperbolic sectors lie in the half-plane \( x > 0 \), and their common boundary trajectory tends to the origin \( O \) along the direction \( \theta = 0 \) as \( t \to +\infty \). If let

\[
\alpha - \beta^2 < 0, \tag{4.2}
\]

then it is easy to show the following properties of the system (4.1):

1. In addition to the origin \( O \), the system (4.1) also have two critical points

\[
O_1 = \left( 0, -\frac{1}{\alpha} \right) \quad \text{and} \quad A = \left( \frac{\beta}{\alpha(\alpha - \beta^2)}, \frac{\beta^2}{\alpha(\alpha - \beta^2)} \right).
\]

\( O_1 \) is a stable node and \( A \) is a saddle point. We shall prove that there must be a trajectory connecting \( O \) and \( A \).

2. Choose \( M_2 = (0, -\frac{1}{3\alpha}) \) and construct the straight line \( y = -\frac{1}{3\alpha} \) through \( M_2 \). It intersects the straight line \( y = \beta x \) at \( Z = (\frac{1}{\alpha\beta}, -\frac{1}{3\alpha}) \). Let \( x_1 = -\frac{1}{3\alpha\beta} - \epsilon \), where \( \epsilon \) is assumed to be small enough \( (0 < \epsilon < \frac{1}{6\alpha\beta}) \) and construct the straight line \( x = x_1 \) through \( P = (x_1, -\frac{1}{3\alpha}) \). It intersects the straight line \( y = k \) at \( Q = (x_i, k_i) \), where

\[
-1 + \frac{1 + 4\alpha^3 x_i^2}{2\alpha} < k_1 < -1 + \frac{1 + \frac{a}{\beta^2}}{2a}.
\]

Let \( M_1 = (0, k_1) \). Therefore, it is easy to see that

\[
\frac{dy}{dt} < 0 \quad \text{for all points on the line segment } M_2 P
\]

and

\[
\frac{dx}{dt} < 0 \quad \text{on the line segment } PQ.
\]

Thus the union \( \gamma_1 = M_2 P \cup PQ \cup QM_1 \) together with two negative semi-trajectories \( \gamma_i(M_i) (i = 1, 2) \) and the critical point \( O \) bound a regular negative parabolic sector \( D_1 \).

3. Choose \( N_2 = (0, -\frac{1}{2\alpha}) \) and construct the straight line \( y = -\frac{1}{2\alpha} \) through \( N_2 \). It intersects the straight line \( y = \beta x \) at \( Z_1 = (\frac{1}{2\alpha\beta}, -\frac{1}{2\alpha}) \). Let

\[
R = \left( -\frac{1}{2\alpha\beta} + \epsilon_1, -\frac{1}{2\alpha} \right)
\]

where \( \epsilon_1 \) is small enough \( (0 < \epsilon_1 < \frac{1}{2\alpha\beta}) \). Clearly, \( \frac{dy}{dt} < 0 \) for all points on the line segment \( N_2 R \).

4. We know from the second expression of (4.1) that \( x = \pm \sqrt{\frac{1+\alpha k^2}{\alpha^2}} \) are the abscissa of those points on the straight line \( y = k \) satisfying \( \frac{dx}{dt} = 0 \). Consider the straight line
\[ y = k' = \frac{\beta^2}{\alpha(\alpha - \beta)} - \varepsilon_2 \] where \( \varepsilon_2 \) is small enough. Then, on the straight line \( y = k' \) for \( x < 0 \), the abscissa satisfying \( \frac{dx}{dt} = 0 \) is as follows:

\[ \dot{x} = \frac{1}{\alpha(\alpha - \beta^2)} \cdot \sqrt{[\beta^2 - \varepsilon_2\alpha(\alpha - \beta^2)][1 - \varepsilon_2(\alpha - \beta^2)]}. \]

Moreover, the abscissa of the intersection point of two straight lines \( y = k' \) and \( y = \beta x \) is as follows:

\[ x^* = \frac{\beta^2 - \alpha \varepsilon_2(\alpha - \beta^2)}{\alpha \beta(\alpha - \beta^2)}. \]

It is not difficult to check that \( \dot{x} < x^* \). Thus, we can take \( x_2 \) such that \( \dot{x} < x_2 < x^* \), and let \( W = (x_2, k') \).

(5) Construct the straight line \( x = \frac{-1}{2\alpha\beta} + \varepsilon_1 \) through \( R \). It intersects the straight line \( y = k' \) at \( L = (-\frac{1}{2\alpha\beta} + \varepsilon_1, k') \). Construct the straight line \( x = x_2 \) through the point \( W \). It intersects the straight line \( y = k_2 \) at \( E = (x_2, k_2) \) where it is assumed that \( k_2 > \frac{-1 + \sqrt{1 + 4\alpha^2x_2^2}}{2\alpha} \).

Clearly, we have \( k_2 > k_1 \). Let \( N_1 = (0, k_2) \). Then, it is easy to verify that

\[ \frac{dy}{dt} < 0 \text{ for all points on the line segment } N_2R \]

\[ \frac{dy}{dt} > 0 \text{ on the line segments } LW \text{ and } EN_1 \]

and

\[ \frac{dx}{dt} > 0 \text{ on the line segment } RL \]

\[ \frac{dx}{dt} < 0 \text{ on the line segment } WE. \]

Thus the union

\[ \gamma_2 = N_2R \cup RL \cup LW \cup WE \cup EN_1 \]

will serve as the simple curve \( N_1N_2 \) in Theorem 3.1. Consider the region \( B \) bounded by \( \gamma_i \) (\( i = 1, 2 \)) together with two line segments \( N_1M_1 \) and \( N_2M_2 \). Noting the fact that the \( y \)-axis is the union of trajectories of the system (4.1) and the properties of three critical points of (4.1), it is easy to see that there are no closed trajectories and singular closed trajectories in \( B \). Further, it is clear that all exit points of \( B \) lying on \( \gamma_2 \) are strict. Therefore, all conditions of Theorem 3.1 are satisfied and it follows that there is in \( B \) a trajectory connecting \( O \) and \( A \).
References


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