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On Cauchy-Kowalevski's Theorem; A Necessary Condition

By

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1. Introduction

We are concerned with the Cauchy-Kowalevski theorem for an equation

$$(1.1) \qquad \partial_t^m u(x, t) = \sum_{j=1}^{m} a_j(x, t; \partial_x)\partial_t^{m-j} u(x, t) + f(x, t), \quad (x, t) \in C^l \times C^1,$$

where the coefficients are assumed holomorphic in a neighborhood of the origin.**) The Cauchy-Kowalevski theorem says that, if

$$(1.2) \qquad \text{order } (a_j) \leqq j,$$

then for any holomorphic Cauchy data $\partial_t^j u|_{t=0} = u_j(x)$ $(0 \leqq j \leqq m-1)$, and for any holomorphic f, given in the neighborhood of the origin, there exists a unique holomorphic solution u of (1.1) in a neighborhood of the origin. In (1.2), order (a_j) means that of a_j *in a neighborhood* of the origin. Our question is the following: Is the condition (1.2) necessary for the Cauchy-Kowalevski theorem? Concerning this, the author showed in [3] the following result. Let q (>1) be the minimum number satisfying

$$\text{order } (a_j) \leqq qj \qquad (1 \leqq j \leqq m),$$

and let $h_j(x, t; \partial)$ be the homogeneous part of order qj of a_j. Then in order that the above Cauchy-Kowalevski theorem hold, it is necessary that

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**) We use the following abbreviations: $\partial_x^\alpha, \partial_t^j$ stand for $\left(\frac{\partial}{\partial x}\right)^\alpha$, $\left(\frac{\partial}{\partial t}\right)^j$ respectively. Furthermore, ∂_x^α will be denoted simply by ∂^α.

(1.3) $$h_j(x, 0: \zeta)\equiv 0.$$

This implies, in particular, when all the terms of order greater than j of $a_j(x, t; \zeta)$ are independent of t, then (1.2) becomes a necessary condition for the validity of the Cauchy-Kowalevski theorem.

Recently M. Miyake investigated this problem [2], and showed that in the case $m=1$, namely,

(1.4) $$\partial_t u=\sum_j b_j(x, t; \partial_x)u+f, \qquad (\text{order } (b_j)=j),$$

the condition $b_j(x, t; \zeta)\equiv 0$ for $j\geqq 2$ is really necessary. So that (1.2) is necessary and sufficient in the case $m=1$. The purpose of this article is to show that when we follow the argument of Miyake together with that of Hasegawa in [1], we arrive at a sharper result than (1.3). Let us explain this. We expand each $a_j(x, t; \zeta)$ appearing in (1.1) in Taylor series in t around the origin. Then the terms appearing on the right hand side take the form:

$$t^n a(x)\, \partial_t^\alpha \partial_t^j \qquad (a(x)\not\equiv 0).$$

To all these terms, we define p (rational number) as the minimum satisfying

$$|\alpha|+p(j-n)\leqq pm.$$

By saying *modified principal part* with weight p of (1.1), we mean all the terms for which the equal sign hold. Our result is:

Theorem. *In order that the Cauchy-Kowalevski theorem hold at the origin, it is necessary that $p\leqq 1$. Accordingly, in particular,*

$$\text{order } (a_j(x, 0; \partial_x))\leqq j$$

is a necessary condition.

2. Preliminaries

To make clear our argument, we treat (1.1) in matrix form. Put $\partial_t^j u=v_{j+1}$ $(0\leqq j\leqq m-1)$. Then (1.1) becomes

(2.1) $$\partial_t v(x, t)=P(x, t; \partial_x)v(x, t)+g(x, t).$$

The Taylor expansion of $P(x, t; \partial_x)$ in t gives

$$P(x, t; \partial_x)=\sum_{j=0}^{\infty} t^j P_j(x; \partial_x), \tag{2.2}$$

where

$$P_0=\begin{bmatrix} 0 & 1 & & \\ & 0 & 1 & \\ & & \ddots & \ddots \\ & & & 1 \\ a_m(x, 0; \partial_x) & \dots & & a_1(x, 0; \partial_x) \end{bmatrix} \tag{2.3}$$

$$P_j=\begin{bmatrix} & 0 & \\ a_{jm}(x; \partial_x) & \dots & a_{j1}(x; \partial_x) \end{bmatrix} \qquad (j\geqq 1)$$

From the definition of the number p, we have

$$\text{order } (a_{jk}(x; \partial))\leqq pj+pk \qquad (j\geqq 0,\ 1\leqq k\leqq m). \tag{2.4}$$

Our purpose is to show that, assuming $p>1$, a formal solution corresponding to an appropriate holomorphic f does not converge in any neighborhood of the origin (assuming the initial data is 0).

Let the modified principal part (with weight p) of P_j be $\mathring{P}_j(x; \partial)$. It is easy to see that there exists an s such that $\mathring{P}_s\neq 0$ but $\mathring{P}_j=0$ for $j>s$. For the foregoing argument, $\mathring{P}_s$ plays an important role. The case $s=0$ can be treated in the similar way as in [3]. Therefore we suppose $s\geqq 1$.

Let

$$P_s(x; \partial)=\begin{bmatrix} & 0 & \\ h_{s1}(x; \partial) & \dots & h_{sm}(x; \partial) \end{bmatrix} \tag{2.5}$$

and $h_{s1}(x; \partial)\equiv\dots\equiv h_{s,i-1}(x; \partial)\equiv 0$. but $h_{si}(x; \partial)\not\equiv 0$, Then, as we shall show at the end of this section, if $h_{si}(0; \zeta)\equiv 0$, then by choosing x_0 near the origin such that $h_{si}(x_0; \zeta)\not\equiv 0$, without loss of generality, we can assume that

$$h_{si}(0; \zeta)\neq 0, \quad \text{for} \quad \zeta\in C^l \qquad (|\zeta|=1).$$

We fix such a ζ once for all.

Next, by choosing j_0 satisfying pj_0=integer, we take $f(x, t)$ as

$$(2.6) \qquad \begin{cases} f(x, t)=\dfrac{t^{j_0}}{j_0!}f_0(x), \\ f_0(x)=\sum_k' c_k \dfrac{\langle\zeta, x\rangle^{pk}}{(pk)!} \end{cases}$$

where $\sum_k'$ means the summation over all positive integers k satisfying pk=integer; c_k will be defined appropriately. To be precise, $|c_k|=(pk)!$ and their arguments are defined recursively. So that $f_0(x)$ is holomorphic in $|x|<1$.

Let

$$(2.7) \qquad v(x, t)\sim\sum_{j=0}^{\infty}\frac{t^j}{j!}v_j(x)$$

be the corresponding formal solution. We assume that $v_0(x)\equiv 0$, i.e. Cauchy data is zero. Then denoting

$$g_0(x)={}^t(0, 0, \dots, 0, f_0(x)),$$

we have $v_0(x)\equiv v_1(x)\equiv\dots\dots\equiv v_{j_0}(x)\equiv 0$. Comparing the coefficient of t^{n-1}, we get

$$(2.8) \qquad \begin{cases} v_n(x)=P_0(x;\partial)v_{n-1}(x)+(n-1)P_1(x;\partial)v_{n-2}(x)+ \\ +(n-1)(n-2)P_2(x;\partial)v_{n-3}+\dots+(n-1)\dots(n-s)P_s(x;\partial)v_{n-s-1}+\dots \end{cases}$$

Let us remark that

$$v_{j+1}(x)=P_0(x;\ \partial)g_0(x).$$

Finally our observation mentioned above (see the assumption with regards to h_{si}) relies on the following proposition which we used in [3]:

Proposition. *Let $\mathcal{O}$ be a complex domain (in $\boldsymbol{C}^{l+1}$) containing the origin. We assume all the coefficients of* (1.1) *belong to $H(\mathcal{O})$, i.e. holomorphic in $\mathcal{O}$. Assume for each $f(x, t)\in H(\mathcal{O})$ there exists a solution $u(x, t)\in H(V_f)$ of* (1.1) *satisfying*

$$\partial_t^j u(x, 0)=0 \text{ for } x\in V_f\cap\{t=0\} \qquad (0\leqq j\leqq m-1),$$

where V_f is a complex domain containing the origin which may depend *on f. Then there exists a fixed complex domain D containing the origin such that for any $f\in H(\mathcal{O})$, there exists always a solution $u(x,t)\in H(D)$ of* (1.1) *with zero Cauchy data.*

Proof. Let

$$D_m=\{(x, t)\in \boldsymbol{C}^{l+1};\ |x_i|<1/m,\ |t|<1/m\}.$$

For any pair (m, n) of positive integers, we define the set $E_{mn}\subset H(\mathcal{O})$ as follows. $f\in E_{mn}$, if and only if there exists a solution $u(x, t)\in H(D_m)$ of (1.1) with zero Cauchy data, satisfying

$$|u(x, t)|\leqq n, \text{ for } (x, t)\in D_m.$$

Then E_{mn} is closed and symmetric. In fact, if $\{f_j\}$ is a sequence of E_{mn}, and $f_j\to f_0$ in $H(\mathcal{O})$, and let $\{u_j\}$ be the corresponding solution. If necessary, by picking a subsequence, we can assume $\{u_{j_p}(x, t)\}$ is a convergent sequence in $H(D_n)$. Then $u_{j_p}(x, t)\to u_0(x, t)$ in $H(D_m)$, and $|u_0(x, t)|\leqq n$ in D_m. Since

$$L(u_{j_p})\equiv\partial_t^m u_{j_p}(x, t)-\sum a_j(x, t;\ \partial_x)\partial_t^{m-j}u_{j_p}(x, t)$$

tends to $L(u_0)$ in $H(D_m)$, we obtain $L(u_0)=f_0$ which proves the closedness of E_{mn}. In view of $H(\mathcal{O})=\bigcup_{m,n} E_{mn}$ by hypothesis, the proposition follows immediately from Baire's category theorem. Q.E.D.

Now let us make precise our hypothesis. First, let

$$\mathcal{O}=\{(x, t);\ |x_i|<\rho,\ |t|<\rho\}$$

where it is assumed $\rho<1/2l$ (recall that l is the dimension of x-space). Then, from the above proposition, we can find a polydisc D, say $D=\{(x, t);\ |x_i|<\rho_0,\ |t|<\rho_0\}$. Now when $h_{si}(0;\zeta)\equiv 0$ for all ζ, and $h_{si}(x;\ \zeta)\not\equiv 0$, we can find $x_0(\in \boldsymbol{R}^l)$ in such a way that denoting $x_0=(x_1^0, \dots, x_n^0)$, it satisfies $|x_i^0|<\min(\rho_0, 1/2l)$, so that we have

$$\mathcal{O}\subset\mathcal{O}'=\{(x, t);\ |x_i-x_i^0|<1/l,\ |t|<1/l\},$$

and that $(x_1^0, \dots, x_n^0, 0)\in D$. This shows that

$$f_0(x)=\sum_k' c_k\frac{\langle\zeta, x-x_0\rangle^{pk}}{(pk)!}\quad (|c_k|=(pk)!),$$

which is holomorphic in $\mathcal{O}'$, is so in $\mathcal{O}$. Thus by hypothesis, the corresponding solution should be holomorphic in a neighborhood V $(\supset D)$ of $(x_0, 0)$.

3. Proof of Theorem

In (2.6), instead of $f_0(x)$ itself, we take simply

$$f_0(x)=\frac{\langle\zeta, x\rangle^{pk}}{(pk)!} \quad (pk=\text{integer}),$$

and consider the coefficients $v_j(x)$ $(j_0\leqq j\leqq j_0+k)$ defined by (2.8). We are concerned with the leading term, i.e. the lowest homogeneous part in x of the h-th component $v_{j,h}(x)$ of $v_j(x)$. Note the following fact. Let

$$\begin{aligned} p(x, \partial)&=\sum_{|\alpha|\leqq n} a_\alpha(x)\,\partial^\alpha=\sum_{|\alpha|=n} a_\alpha(x)\,\partial^\alpha+\sum_{|\alpha|<n} a_\alpha(x)\,\partial^\alpha \\ &=p_n(x, \partial)+q(x, \partial). \end{aligned}$$

Then for $j\geqq n$,

$$p(x, \partial)\frac{\langle\zeta, x\rangle^j}{j!}=p_n(0, \zeta)\frac{\langle\zeta, x\rangle^{j-n}}{(j-n)!}+\cdots\cdots$$

where the rest term on the right hand side is analytic function of vanishing order $\geqq j-n+1$. In view of this, we see that

$$v_{j,h}(x)=\alpha_{j,h}\langle\zeta, x\rangle^{\nu(j,h)}/\nu(j, h)!+\cdots\cdots$$

where $\nu(j, h)=pk-p(j-j_0)+p(m-h)$, and the rest term is of vanishing order $\geqq\nu(j, h)+1$.

Taking account of that, (2.8) gives a recurrence formula for $\alpha_{j,h}$. In fact, let

$$P_0(\zeta)=\mathring{P}_0(0; \zeta)=\begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ h_{01}(0; \zeta) & \cdots\cdots & & & h_{0m}(0; \zeta) \end{bmatrix},$$

$$P_j(\zeta)=\mathring{P}_j(0; \zeta)=\begin{bmatrix} & 0 & \\ h_{j1}(0; \zeta) & \cdots\cdots & h_{jm}(0; \zeta) \end{bmatrix} \quad (j\geqq 1),$$

where $h_{jk}(0;\,\zeta)$ are the terms of the modified principal part with weight p (see the definition in the Introduction), and let

$$\alpha_n={}^t(\alpha_{n,1},\dots,\alpha_{n,m}).$$

Then

$$\text{(3.1)}\quad \begin{cases} \alpha_n=P_0(\zeta)\alpha_{n-1}+(n-1)P_1(\zeta)\alpha_{n-2}+(n-1)(n-2)P_2(\zeta)\alpha_{n-3}+ \\ \qquad +\dots+(n-1)(n-2)\dots(n-s)P_s(\zeta)\alpha_{n-s-1}, \end{cases}$$

where

$$\alpha_j=0 \text{ for } j<j_0 \text{ and } \alpha_{j_0}={}^t(0,\,0,\,\dots,\,0,\,1).$$

Hereafter we denote

$$h_{sj}(0;\,\zeta)=h_j \qquad (1\leqq j\leqq m).$$

Then by hypothesis, $h_1=h_2=\dots=h_{i-1}=0$, but $h_i\neq 0$. For convenience of the foregoing argument, let us denote

$$\alpha'_n={}^t(\alpha_{n,i},\ \alpha_{n,i+1},\ \dots,\ \alpha_{n,m}),$$

and

$$|\alpha_n|=\sum_{j=1}^{m}|\alpha_{n,j}|\,;\quad |\alpha'_n|=\sum_{j=i}^{m}|\alpha_{n,j}|.$$

Next we choose a δ $(0<\delta<1)$ once for all in such a way that

$$\text{(3.2)}\qquad \max_{j>i}|h_j|\frac{\delta}{1-\delta}\leqq\frac{1}{2}|h_i|.$$

Then for the sequence $\{\alpha_n\}$ we have the following lemma:

Lemma. *If j_0 is chosen large, then there exists an infinite subsequence $\{\alpha_{n_p}\}$ of $\{\alpha_n\}$ $(n\geqq j_0)$ satisfying the following increasing law (in the wider sense):*

$$|\alpha'_{n_p}|\geqq\delta|\alpha'_{n_{p-1}}| \qquad (p=2,\,3,\,\dots),$$

moreover we can assume that

$$n_p-n_{p-1}\leqq s+1.$$

Proof. To define $\{n_p\}$, we proceed as follows: If n_p is defined, then n_{p+1} is defined as the minimum number m $(>n_p)$ satisfying

$$|\alpha'_m|\geqq\delta|\alpha'_{n_p}|. \tag{3.3}$$

Accordingly, let $n_p=n$, then it suffices to prove the following fact: If

$$|\alpha'_{n+1}|,\ |\alpha'_{a+2}|,\ \dots,\ |\alpha'_{n+s}|<\delta|\alpha'_n|, \tag{3.4}$$

then,

$$|\alpha'_{n+s+1}|\geqq\delta\,|\alpha'_n|.$$

Let us consider the last, i.e. the m-th component of α_{n+s+1}. In view of (3.1), it amounts to consider those of $P_{s-i}(\zeta)\alpha_{n+i}$ $(0\leqq i\leqq s)$. As we can observe, the case where $i=m$ is easy, so we argue as $i<m$. Denote the m-th component of $P_s(\zeta)\alpha_n$ by $(P_s\alpha_n)_m$.

$$(P_s(\zeta)\alpha_n)_m=h_i\alpha_{n,i}+\sum_{j>i}h_j\,\alpha_{n,j}.$$

Thus,

$$|(P_s(\zeta)\alpha_n)_m|\geqq|h_i|\,|\alpha_{n,i}|-\max_{j>i}|h_j|\sum_{j>i}|\alpha_{n,j}|. \tag{3.5}$$

On the other hand, from (2.8) we see that, for general n,

$$\alpha_{n,j}=\alpha_{n-1,j+1}\qquad(1\leqq j\leqq m-1). \tag{3.6}$$

In fact, all the entries of $P_1(\zeta),\ \dots,\ P_s(\zeta)$ are zero except the mth row, so that the components $\alpha_{n,j}$ $(1\leqq j\leqq m-1)$ of α_n can be defined simply by $\alpha_n=P_0(\zeta)\alpha_{n-1}$. By hypothesis (3.4),

$$\sum_{j=i}^{m}|\alpha_{n+1,j}|<\delta\sum_{j=i}^{m}|\alpha_{n,j}|.$$

Next, by (3.6),

$$\begin{aligned}\sum_{j=i+1}^{m}|\alpha_{n,j}|=\sum_{j=i+1}^{m}|\alpha_{n+1,j-1}|&=\sum_{j=i}^{m-1}|\alpha_{n+1,j}|\\ \leqq\sum_{j=i}^{m}|\alpha_{n+1,j}|&<\delta\sum_{j=i}^{m}|\alpha_{n,j}|.\end{aligned}$$

Thus,

$$(1-\delta)\sum_{j=i+1}^{m}|\alpha_{n,j}|<\delta|\alpha_{n,i}|.$$

This implies, from (3.5) and (3.2),

$$|(P_s(\zeta)\alpha_n)_m|\geqq|h_i|\,|\alpha_{n,i}|-\max_{j>i}|h_j|\frac{\delta}{1-\delta}|\alpha_{n,i}|\geqq\frac{1}{2}|h_i|\,|\alpha_{n,i}|.$$

Further, since

$$|\alpha'_n|=|\alpha_{n,i}|+\sum_{j>i}|\alpha_{n,j}|\leqq|\alpha_{n,i}|\left(1+\frac{\delta}{1-\delta}\right),$$

i.e. $|\alpha_{n,i}|\geqq(1-\delta)|\alpha'_n|$, the above relation can be written as

$$(3.7)\qquad |(P_s(\zeta)\alpha_n)_m|\geqq\frac{1-\delta}{2}|h_i|\,|\alpha'_n|.$$

Finally let us consider the last component of $P_{s-1}(\zeta)\alpha_{n+1}$, $P_{s-2}(\zeta)\alpha_{n+2}$, ..., $P_0(\zeta)\alpha_{n+s}$. First, from our process of choosing $\{n_p\}$ (defined at the beginning), we have

$$(3.8)\qquad |\alpha'_{n-j}|\leqq\frac{1}{\delta^j}|\alpha'_n|\qquad (j=1, 2, \ldots).$$

In fact, it holds that $|\alpha'_{n_{p-1}}|\leqq\frac{1}{\delta}|\alpha'_{n_p}|$, $|\alpha'_{n_{p-2}}|\leqq\frac{1}{\delta}|\alpha'_{n_{p-1}}|\leqq\frac{1}{\delta^2}|\alpha'_{n_p}|$, and so on. Further for $n_{q-1}<\nu<n_q$, it holds $|\alpha'_\nu|<|\alpha'_{n_q}|$.

Next, from (3.6), for any h $(1\leqq h\leqq i-1)$, we have

$$\alpha_{n+j,h}=\alpha_{n+j-1,h+1}=\alpha_{n+j-2,h+2}=\cdots .$$

This gives together with (3.8)

$$(3.9)\qquad |\alpha_{n+j,k}|\leqq\frac{1}{\delta^m}|\alpha'_n|\qquad (0\leqq j\leqq s),$$

and this is of course true for any k $(1\leqq k\leqq m)$ (see (3.4)).

Thus we have

$$(3{\cdot}10)\qquad \sum_{\nu=1}^{s}|(P_{s-\nu}(\zeta)\alpha_{n+\nu})_m|\leqq sm\cdot\max_{i,j}|h_{ij}(0;\zeta)|\frac{1}{\delta^m}|\alpha'_n|\equiv K|\alpha'_n|,$$

where

$$K=\frac{sm}{\delta^m}\max_{i,j}|h_{ij}(0;\,\zeta)|. \tag{3.11}$$

Finally from (3.1), (3.7) and (3.10), we obtain

$$\begin{aligned}|\alpha_{n+s+1,m}|&\geqq(n+s)\,(n+s-1)\dots(n+1)|(P_s(\zeta)\alpha_n)_m|\\&\quad-(n+s)\dots(n+2)\sum_{\nu=1}^{s}|(P_{s-\nu}(\zeta)\alpha_{n+\nu})_m|\\&\geqq(n+s)\dots(n+1)\left\{\frac{1-\delta}{2}|h_i|\,|\alpha'_n|-\frac{1}{n+1}K|\alpha'_n|\right\}.\end{aligned}$$

Now we define j_0 in such a way that

$$\frac{1-\delta}{4}|h_i|j_0\geqq\max(1,\,K). \tag{3.12}$$

Then we have $|\alpha_{n+s+1,m}|\geqq|\alpha'_n|$. This completes the proof. Q.E.D.

Let us return to (2.6). The above lemma gives the following result: there exist i_0, j ($0\leqq i_0\leqq s$, $1\leqq j\leqq m$) such that, if we write

$$v_{n+j_0-i_0,j}(x)=c_n\,\beta_n\frac{\langle\zeta,\,x\rangle^k}{k!}+\varphi_n(x)+\psi_n(x),$$

where $k=pi_0+p(m-j)$, $\psi_n(x)$ being of vanishing order strictly greater than k, and $\varphi_n(x)$ is determined by $\{c_i\}$ for $i<n$, then, *there exists an infinite subsequence of n satisfying*

$$|\beta_n|\geqq\delta'^n\qquad(\delta'>0) \tag{3.13}$$

for a fixed δ'.

Let us explain this. First we choose i_0 in such a way that there exists an infinite subsequence $\{n_p\}$ stated in the above lemma, which is congruent to $1-i_0$ modulo the denominator of p. Next, j is chosen in such a way that, for this subsequence, say $\{\alpha'_{n_p}\}$, we have $|\alpha'_{n_p,j}|\geqq\frac{1}{m}|\alpha'_{n_p}|$. Further let us decompose (pn=integer),

$$f_0(x)=c_n\frac{\langle\zeta,\,x\rangle^{pn}}{(pn)!}+\sum_{k<n}c_k\frac{\langle\zeta,\,x\rangle^{pk}}{(pk)!}+\sum_{k>n}{}'\,c_k\frac{\langle\zeta,\,x\rangle^{pk}}{(pk)!}.$$

Since the correspondence

$$f_0(x)\longrightarrow v_n(x)$$

is linear (see (3.1)), for the study of the structure of $v_n(x)$ we can consider the three terms separately. Now we see easily that the part of $v_{n+j_0-i_0,j}(x)$ corresponding to the third term in the above decomposition, is of vanishing order$\geqq pi_0+p(m-j)+p$, hence this is greater than $k+1$.

Thus we obtain taking account of $|\zeta|=1$,

$$\langle\bar{\zeta}, \partial\rangle^k v_{n+j_0-i_0,j}(x)|_{x=0}=c_n \beta_n+\langle\bar{\zeta}, \partial\rangle^k \varphi_n(x)|_{x=0}. \tag{3.14}$$

Now we define c_n by

$$c_n=(pn)!\ e^{i\theta_n}, \tag{3.15}$$

where θ_n is fixed in such a way that $e^{i\theta_n}\beta_n$ and $\langle\bar{\zeta}, \partial\rangle^k \varphi_n(x)|_{x=0}$ have the same argument. It follows from (3.13) that for an appropriate subsequence of *n the left hand side of* (3.14) *is greater than, in absolute value,* $(pn)!\ \delta'^n$.

Now we return to the formal solution (2.7), and consider

$$\langle\bar{\zeta}, \partial\rangle^k v(x, t)|_{x=0} \sim \sum_{n\geqq 0} \frac{t^n}{n!} \langle\bar{\zeta}, \partial\rangle^k v_n(x)|_{x=0}.$$

There are infinitely many integers of the form $n+j_0-i_0$ such that their j-th component are greater than in absolute value

$$\frac{1}{(n+j_0-i_0)!}|c_n \beta_n|\geqq \frac{(pn)!}{(n+j_0-i_0)!}\delta'^n \underset{n\to\infty}{\sim} A\, c_0^n\, n^{(p-1)n},$$

where A and c_0 are appropriate positive constants. Since $p>1$, the above series is never convergent for any $t(\neq 0)$. This completes the proof of Theorem in the Introduction.

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