Exposed Operators in $\mathcal{B}(C(X), C(Y))$

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A point $q_0$ in a convex set $Q$ is exposed if there exists a bounded linear functional $\xi$ such that $\xi(q_0) > \xi(q)$ for all $q \in Q \setminus \{q_0\}$. Characterizations of exposed points of the unit ball and the positive part of the unit ball of $\mathcal{B}(C(X), C(Y))$ are given. We describe the set of strongly exposed points. We also consider exposed operators on $L^\infty$- and $L^1$-spaces.

Key words: Exposed points, space of continuous functions, operators, Nice operators, strongly exposed operators

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1. Introduction

Let $B_1, B_2$ be Banach spaces. By $\mathcal{B}(B_1, B_2)$ there is denoted the Banach space of all bounded linear operators from $B_1$ to $B_2$. An operator $T \in \mathcal{B}(B_1, B_2)$ is called a contraction if $\|T\| \leq 1$.

Throughout this paper we assume that $X$ and $Y$ are non-empty compact Hausdorff topological spaces. As usual, we denote by $C(X)$ the Banach space of all real-valued (or complex) continuous functions on $X$ with supremum norm. Following Morris and Phelps [23], we call an contraction $T \in \mathcal{B}(C(X), C(Y))$ nice, if its adjoint operator $T^*$ takes Dirac measures on $Y$ into extreme points of the unit ball of $C(X)^*$. It is not difficult to see that every nice operator is an extreme contraction (extreme point of the unit ball). Note that any element of $C(X)^*$ can be identified both as a linear functional and a measure. Moreover, the set of all extreme points of the unit ball of $C(X)^*$ coincides with the set $\{a\delta_x: |a| = 1, x \in X\}$, where $\delta_x$ denotes Dirac measure (point mass) at $x \in X$ (see [2]). Thus $T \in \mathcal{B}(C(X), C(Y))$ is nice if and only if there exists a function $r \in C(Y)$ with $|r| = 1$ and a continuous map $\varphi: Y \to X$ such that $(Tf)(y) = r(y)f(\varphi(y))$, for all $f \in C(X)$ and $y \in Y$.

Each extreme contraction in $\mathcal{B}(C(X), C(Y))$ is nice in the following cases:

1. $X$ is metrizable (see [4]).
2. $X$ is Eberlein compact, $Y$ is metrizable (see [1]).
3. $X$ is dispersed (see [27]).
4. $Y$ is extremally disconnected (see [27]; also [8, 18]).

It should be pointed out that Sharir [28, 29] has given counterexamples (also [12]). Extreme operators have been studied by many authors. The first theorem of this type was given by A. and C. Ionescu Tulcea [16, 17]. Consider positive operators (an operator $T$ is called positive if $Tf \geq 0$ for all $f \geq 0$). An operator $T \in \mathcal{B}(C(X), C(Y))$ is an extreme positive contraction (extreme point of the positive part of the unit ball of operators) if and only if there exists a clopen (closed and open) set $Z \subseteq Y$ and a continuous map $\varphi: Z \to X$ such that $(Tf)(y) = 0$ if $y \in Z$ and $(Tf)(y) = f(\varphi(y))$ if $y \notin Z$ (see [6, 24], also [26: III/§9]).

Recall that a point $q_0$ in a convex set $Q$ is exposed if there exists a bounded real linear functional $\xi$ such that $\xi(q_0) > \xi(q)$ for all $q \in Q \setminus \{q_0\}$. An exposed point $q_0 \in Q$ is called strongly exposed if for any sequence $(q_n) \subseteq Q$ the condition $\xi(q_n) \to \xi(q_0)$ implies $q_n \to q_0$. Obviously each exposed point is extreme.

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The purpose of this paper is to study exposed and strongly exposed points of the unit ball of $\mathcal{B}(C(X), C(Y))$. We also consider exposed operators acting on $L^1$- and $L^\infty$- spaces. Note that exposed operators in $L^p$-spaces are considered in [13,14]. We should mention here that P. Greim also obtained results in this direction for Bochner $L^p$-spaces [9 - 11].

We say that a compact Hausdorff space $X$ carries a strict positive measure if there exists a strictly positive Radon measure $\mu$ on $X$ (i.e. $\mu(U) > 0$ for all non-empty open subsets $U$ of $X$).

The problem of characterization of spaces $X$ which carry a strictly positive measure has been studied by many authors (see, e.g., [3,7,15,21,22]). In particular Kelley [19] introduced the notion of intersection numbers of a collection of subsets to give the characterization of spaces which carry a strictly positive measure. It should be pointed out that in the case of a compact Hausdorff space the problem mentioned above is equivalent to the problem of existence of a finitely additive strictly positive measure. Note that $C(X)$ carries a strictly positive functional if and only if its dual $C(X)^*$ contains a weakly compact total subset (see [24: Theorem 6.5.b]). We refer the reader to [5: Chapter 6] for a summary of those and related results.

2. Exposed points in $\mathcal{B}(C(X), C(Y))$

We recommend to begin with a general sentence.

**Theorem 1:** Let $Y$ carry a strictly positive measure and suppose that

(i) $X$ is metric or (ii) $Y$ is extremally disconnected.

Then each extreme point of the unit ball of $\mathcal{B}(C(X), C(Y))$ is exposed.

**Proof:** Let $\mu$ be a strictly positive measure on $Y$ with $\mu(Y) = 1$. Let $T_0$ be nice (= extreme), i.e. there exist $r_0 \in C(Y)$ and a continuous map $\Phi: Y \to X$ such that $(T_0f)(y) = r_0(y)f(\Phi(y))$, $|r_0| = 1$. For $T \in \mathcal{B}(C(X), C(Y))$ and $y \in Y$ we denote by $m_y^T$ the measure defined by the equality $(Tf)(y) = \int_X f(x)m_y^T$ for all $f \in C(X)$. This is a signed regular Borel measure on $X$ with total variation $\|m_y^T\| \leq \|T\|$. In fact $m_y^T = T^*\delta_y$. If $T$ is a metric compact Hausdorff space. For $n \in \mathbb{N}$ and $y \in Y$ we define by $h_n,y(x) = r_0(y)\max(1 - nd(x, \Phi(y)), 0)$ an element of $C(X)$. The map $h_n,: Y \to h_n,y \in C(X)$ is continuous, so for every operator $S \in \mathcal{B}(C(X), C(Y))$ the function $y \to \langle Sh_n,y(y) \rangle$ is continuous (as an element of $C(Y)$). Now we define a linear functional $\xi_i$ on $\mathcal{B}(C(X), C(Y))$ by

$$\xi_i(S) = \sum_{n=1}^{\infty} \frac{1}{n} \int_Y (Sh_n,y)(y)dm_y(y) \quad (S \in \mathcal{B}(C(X), C(Y))).$$

If $\|S\| \leq 1$, then $|Sh_n,y(y)| \leq 1$ and $\|\xi_i\| \leq 1$. Suppose that $\xi_i(S_0) = 1 = \xi_i(T_0)$ for some contraction $S_0$. Then $\int (S_0h_n,y)(y)dm_y(y) = 1$ for all $n \in \mathbb{N}$. Since the map $y \to \langle S_0h_n,y(y) \rangle$ is continuous with $|(S_0h_n,y)(y)| \leq 1$ we get $\langle S_0h_n,y(y) \rangle = \langle h_n,y, S_0^*\delta_y \rangle = 1$ for $y \in Y$. Hence $S_0^*\delta_y = r_0(y)\delta_{\Phi(y)}$, i.e. $S_0 = T_0$, what show that $T_0$ is exposed.

Now suppose that $Y$ is extremally disconnected. Then $C(Y)$ is an order complete AM-space with unit [26: Section II.7.7] and $\mathcal{B}(C(X), C(Y))$ is a Banach lattice [26: Section IV.1.5.]. Therefore for every contraction $S \in \mathcal{B}(C(X), C(Y))$ there exist positive contractions $S_+$ and $S_-$ such that $S = S_+ - S_-$. Then $m_y^{S_+} = m_y^{S_-}$ and $m_y^{S_-}$ weakly $\ast$ continuous. Since for arbitrary nets $y_\alpha \to y_0$ in $Y$ and $\beta_\alpha \to \beta_0$ in $\mathbb{R}$ with $\beta_\alpha \in [0, m_y^{S_+}(\langle \varphi(y_\alpha) \rangle)]$ the condition $m_y^{S_+} - \beta_\alpha \delta_{\varphi(y_\alpha)} \geq 0$ implies that $m_y^{S_-} - \beta_0 \delta_{\varphi(x_0)} \geq 0$ and the sets \{ $y : m_y^{S_+}(\langle \varphi(y) \rangle) \geq a$ \} are closed.
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for all \( a \in \mathbb{R} \). The same we have for \( m_yS \). Therefore \( \int_0^\infty (y) m_yS(\{\varphi(y)\}) \, d\mu(y) \) exists for every \( S \in \mathcal{B}(C(X), C(Y)) \).

We define a linear functional \( \xi_2 \) on \( \mathcal{B}(C(X), C(Y)) \) by

\[
\xi_2(S) = \int_Y g_0^{-1}(y) m_yS(\{\varphi(y)\}) \, d\mu(y) \quad (S \in \mathcal{B}(C(X), C(Y))).
\]

For a contraction \( S \in \mathcal{B}(C(X), C(Y)) \) and an element \( y \in Y \) we have \( \|m_yS\| \leq 1 \). Thus \( \xi_2(S) \leq \mu(Y) = 1 \). Moreover \( \xi_2(S_0) = 1 \) for some contraction \( S_0 \in \mathcal{B}(C(X), C(Y)) \).

We have \( \|m_yS_0\| \leq 1 \). Thus \( m_yS_0(\{\varphi(y)\}) = r_0(y) \) \( \mu \)-a.e. Hence \( m_yS_0 = r_0(y) \delta_\varphi(y) \) \( \mu \)-a.e. Therefore by a continuity argument and the fact that \( \{ y : m_yS_0(\{\varphi(y)\}) = r_0(Y) \} \) is closed we obtain \( (S_0f)(y) = r_0(y)f(\varphi(y)) \) for all \( y \in Y \). Thus \( S_0 = T_0 \), i.e. \( T_0 \) is exposed by \( \xi_2 \) in the unit ball of \( \mathcal{B}(C(X), C(Y)) \).

**Theorem 2:** Let \( Y \) carry a strictly positive measure on \( Y \). Then each extreme point of the positive part of the unit ball of \( \mathcal{B}(C(X), C(Y)) \) is exposed.

**Proof:** Let \( \mu \) be a strictly positive measure on \( Y \) and \( T_0 \) an extreme positive contraction. Then there exists a clopen set \( Z \subseteq Y \) and a continuous map \( \varphi : Z \to X \) such that \( (T_0f)(y) = 0 \) if \( y \not\in Z \) and \( (T_0f)(y) = f(\varphi(y)) \) if \( y \in Z \). Now we define a linear functional \( \xi \) by

\[
\xi(S) = \int_Z m_yS(\{\varphi(y)\}) \, d\mu(y) - \int_Z c(S \mathcal{B}X)(y) \, d\mu(y) \quad (S \in \mathcal{B}(C(X), C(Y))).
\]

This functional exposes \( T_0 \). Indeed, for a positive contraction \( S \) we have \( \xi(S) \leq \mu(Z) = \xi(T_0) \).

Suppose \( \xi(S_0) = \mu(Z) \) for some positive contraction \( S_0 \). Then \( m_yS_0(\{\varphi(y)\}) = 1 \) for \( y \in Z \) and \( (S_0f)(y) = 0 \) for \( y \in Z^c \). Using the same arguments as in the proof of Theorem 1 we have \( (S_0f)(y) = f(\varphi(y)) \) for \( y \in Z \). If \( 0 \leq f \leq 1 \), then \( 0 \leq S_0f \leq S_0I \), so \( (S_0f)(y) = 0 \) for \( y \in Z^c \). Therefore \( S_0 = T_0 \), i.e. \( T_0 \) is exposed by \( \xi \).

3. Strongly exposed operators

Now we consider the strongly exposed points of the unit ball and the positive part of the unit ball of \( \mathcal{B}(C(X), C(Y)) \).

**Theorem 3:** Let \( Y \) carry a strictly positive measure and \( X \) be metric or extremally disconnected.

(a) If \( \text{card} Y < \infty \), then all extreme points of the unit ball of the space \( \mathcal{B}(C(X), C(Y)) \) are strongly exposed.

(b) If \( \text{card} Y = \infty \), then there are no strongly exposed points in the unit ball of the space \( \mathcal{B}(C(X), C(Y)) \).

**Proof:** (a) Let \( Y = \{y_1, y_2, \ldots, y_n\} \), \( n \in \mathbb{N} \) and \( T_0 \) be an extreme contraction. Then \( (T_0f)(y_j) = r(y_j)f(\varphi(y_j)) \), where \( |r(y_j)| = 1 \). Put \( \xi(S) = \sum_{j=1}^n r(y_j) m_{y_j}S(\{\varphi(y_j)\}) \), where \( m_{y_j}S \) denotes the measure on \( X \) defined in the proof of Theorem 1. Obviously \( \xi \) exposes \( T_0 \). Suppose that \( \xi(S_k) \to \xi(T_0) = n \) for some sequence of contractions \( S_k \in \mathcal{B}(C(X), C(Y)) \). Then \( \|m_{y_j}S_k\| \leq 1 \) and \( \|m_{y_j}S_k(\{\varphi(y_j)\}) \to r(y_j) \) as \( k \to \infty \). Thus

\[
\|m_{y_j}S_k - r(y_j)\delta_{\varphi(y_j)}\| \leq \|m_{y_j}S_k - r(y_j)\delta_{\varphi(y_j)}structured\{\varphi(y_j)\} = \|m_{y_j}S_k\{\varphi(y_j)\}\| \to 0
\]

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as \( k \to \infty \). Now we obtain
\[
\|S_k - T_0\| = \sup_{\|f\| = 1} \sup_{j \leq n} \left| \int_X f \left( m_{y_j}^k - r(y_j) \delta_{\varphi(y_j)} \right) \right| \\
\leq \sup_{\|f\| = 1} \left( \sup_{j \leq n} \left\| m_{y_j}^k - r(y_j) \delta_{\varphi(y_j)} \right\| \right) \to 0 \quad \text{as} \quad k \to \infty.
\]
Thus \( T_0 \) is strongly exposed.

(b) Suppose now that \( \text{card} \ Y = \infty \). Let \( T_0 \) be exposed by a functional \( \xi \), i.e. \( \xi(S) \leq \xi(T_0) = 1 \) for all contractions \( S \). There exists a sequence \( \left( U_j \right)_{j=1}^\infty \) of disjoint non-empty open subsets of \( Y \). Let \( g_j \in C(Y) \) be such that \( \|g_j\| = 1 \), and \( \text{supp} \ g_j \subset U_j \) for all \( j \in \mathbb{N} \). Now we define operators \( R_j \) by \( R_j f = g_j T_0 f \). Put \( \gamma_j = \xi(R_j) \) and let \( k \in \mathbb{N} \). The operator \( \sum_{j=1}^k R_j \) is a contraction. Thus \( \gamma_j = \xi(\sum_{j=1}^k R_j) \leq 1 \). Therefore \( \lim_j \gamma_j = 0 \). Consider the operators \( T_j = T_0 - R_j \). We have \( \|T_0 - T_j\| = 1 \), i.e. \( \{T_j\} \) does not converge to \( T_0 \). But \( \xi(T_j) = \xi(T_0) - \xi(R_j) = 1 - \gamma_j \to 1 = \xi(T_0) \) as \( j \to \infty \). Thus \( T_0 \) is not strongly exposed.

**Theorem 4:** The following statements are true.

(a) If \( \text{card} \ Y < \infty \), then all extreme points of the positive part of unit ball of the space \( \mathcal{U}(C(X), C(Y)) \) are strongly exposed.

(b) If \( \text{card} \ Y = \infty \), then there are no strongly exposed points in the positive part of the unit ball of the space \( \mathcal{U}(C(X), C(Y)) \).

**Proof:** Let \( T_0 \) be a positive contraction in \( \mathcal{B}(C(X), C(Y)) \). Then \( (Tf)(y) = 0 \) for \( y \notin Y \) and \( (Tf)(y) = f(\varphi(y)) \) for \( y \in Y \).

(a) Suppose that \( \text{card} \ Y < \infty \). We define a functional \( \xi_0 \) by
\[
\xi_0(S) = \sum_{j \in \gamma} m_{y_j}^{S}(\varphi(y_j)) - \sum_{j \in \gamma} (S 1_{\gamma})(y_j).
\]
This functional exposes \( T_0 \) since \( \xi_0(S) \leq \text{card} \ Y = \xi_0(T_0) \). Suppose now that \( \xi_0(S_k) \to \xi_0(T_0) \) as \( k \to \infty \), for some sequence of positive \( S_k \). We have
\[
\|m_{y_j}^{S_k} - \delta_{\varphi(y_j)}\| \to 0 \quad \text{for} \quad y_j \in Y \quad \text{and} \quad (S_k 1_{\gamma})(y_j) \to (T_0 1_{\gamma})(y_j) \quad \text{as} \quad j \to \infty.
\]
Therefore \( \|S_k - T_0\| \to 0 \) as \( n \to \infty \), i.e. \( T_0 \) is strongly exposed.

(b) Suppose that a functional \( \xi \) exposes a positive contraction \( T_0 \), i.e. \( \xi(S) \leq \xi(T_0) = 1 \) for all positive contractions \( S \). If \( \text{card} \ Y = \infty \), then using arguments from the proof of Theorem 4/(b) we get that \( T_0 \) is not strongly exposed. Consider now the case \( \text{card} \ Y = \infty \). Let \( \left( U_j \right)_{j=1}^\infty \) be a family of disjoint non-empty open sets and let \( g_j \) be such that \( 0 \leq g_j \leq 1 \), supp \( g_j \subset U_j \), \( \|g_j\| = 1 \). Fix \( x_0 \in X \). We define operators \( R_j \) by \( R_j f(x) = g_j(x) f(x_0) \). Put \( \gamma_j = \xi(R_j) \) and let \( k \in \mathbb{N} \). Since \( \sum_{j=1}^k R_j \) is a positive contraction, using the same arguments as in the proof of Theorem 3/(b) we obtain \( \gamma_j \to 0 \) and \( \xi(T_j) \to \xi(T_0) \) as \( j \to \infty \), where \( T_j = T_0 + R_j \), though \( \|T_j\| \leq 1 \), \( T_j \geq 0 \), and \( \|T_j - T_0\| \leq 1 \). Thus \( T_0 \) is not strongly exposed.

**Theorem 5:** If \( Y \) does not carry a strictly positive measure, then there are not exposed points in the unit ball and in the positive part of the unit ball of \( \mathcal{U}(C(X), C(Y)) \).

**Proof:** First consider the case of the whole unit ball. Suppose that \( \xi_0 \) exposes an extreme contraction \( T_0 \in \mathcal{U}(C(X), C(Y)) \). We define a functional \( m \) on \( C(X) \) by \( m(h) = \xi_0(hT_0) \), \( h \in C(X) \). Suppose that there exists a non-zero \( h_0 \), \( 0 \leq h_0 \leq 1 \) and \( m(h_0) = 0 \). Then \( \xi_0(1 - h_0)T_0 = m(1 - h_0) \geq m(1) = \xi_0(T_0) \). Since \( \|1 - h_0\|T_0 \leq 1 \), we have \( (1 - h_0)T_0 = T_0 \). Fix \( x_0 \in X \). Because \( \|1 - h_0\|T_0 f \)}
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The operator $(1 - h_0)T_0$ is not extreme. This contradictions proves that $m(h) > 0$ for all $h \in C(Y)$ with $0 \leq h \leq 1$. Therefore if there exists an exposed point in the unit ball, then $Y$ carries a strictly positive measure, what ends the proof for the unit ball.

Now consider the positive part of the unit ball. Suppose that a functional $\xi_0$ exposes a positive contraction $T_0$. The operator $T_0$ is of the form $T_0(f X y) = f(\varphi(y))$ for $y \in Z$. Using arguments presented in the first part of the proof one can see that the clopen set $Z$ carries a strictly positive measure.

Let $x_0 \in X$ and put $R = I_Z \xi(f(x_0))$, $f \in C(X)$. We define a functional $n$ on $C(Z^\circ)$ by $n(h) = -\xi_0(h R)$, $h \in C(Z^\circ)$.

Let $0 \leq h \leq 1$ and $h \neq 0$. Then because $T_0 + h R$ is a positive contraction and $h R \neq 0$ we have $\xi_0(T_0 + h R) = \xi_0(T_0) - n(h)$, so $n(h) > 0$. Therefore $n$ is a strictly positive measure on $Z^\circ$.

4. The case of $L^\infty$- and $L^1$-spaces

Let $(Q, \mathcal{B}, \mu)$ be a $\sigma$-finite measure space. Denote by $L^\infty(\mu)$ the space of all essentially measurable functions on $(Q, \mu)$, with essential supremum norm. The space $L^\infty(\mu)$ is the dual of the AL-space $L^1(\mu)$, and is isomorphic to $C(X)$, where $X$ is the Stone representation space of $\beta/N$ ($N$ denotes the ideal of measure zero sets). In this case the space $X$ must be hyperstonian (see [26: Chap. II, Sec. 9]). Thus $X$ is also Stonean (extremally disconnected). Since $\mu$ is $\sigma$-finite, there exists a strictly positive $1 \in L^1(\mu)$. Hence $X$ carries a strictly positive measure.

Let $(Q_i, \mathcal{B}_i, \mu_i)$ be $\sigma$-finite measure spaces, $i = 1,2$. Consider now extreme operators in $\mathcal{B}(L^\infty(\mu_1), L^\infty(\mu_2))$. We can identify this space with the space $\mathcal{B}(C(X), C(Y))$, where $X$ and $Y$ are hyperstonian spaces. Note that the representation of an extreme operator in $\mathcal{B}(L^\infty(\mu_1), L^\infty(\mu_2))$ by means of a measurable transformation $\varphi$ is not always possible (see [18: p. 152]). The extreme positive contractions in the space $\mathcal{B}(L^\infty(\mu_1), L^\infty(\mu_2))$ can be characterized as operators which carry characteristic functions, or equivalently, which are multiplicative (see [24: Theorem 2.2]). The set of extreme contractions in $\mathcal{B}(L^\infty(\mu_1), L^\infty(\mu_2))$ coincides with the set of all lattice homomorphisms taking the function $1$ into itself, multiplied by functions of absolute value one (see [18,20]). Using Theorem 2 and 3 we obtain the following.

**Theorem 6:** Extreme positive contractions and extreme contractions in $\mathcal{B}(L^\infty(\mu_1), L^\infty(\mu_2))$ are exposed. Moreover the exposed operators are strongly exposed if and only if $L^\infty(\mu_2)$ is finite-dimensional.

Let us consider extreme operators in $\mathcal{B}(L^1(\mu_1), L^1(\mu_2))$. The extreme contractions can be characterized as those operators whose adjoints are extreme contractions in $\mathcal{B}(L^\infty(\mu_2), L^\infty(\mu_1))$ (see [18]). As we mentioned above, in general extreme operators cannot be represented by measurable transformations. But in some cases this is possible, for example, if $\mu_2$ is a $\sigma$-finite Borel measure on $R$ (see [18: Theorem 2]). Also extreme positive contractions are characterized by duality.

**Theorem 7:** Let $\mu_2$ be a $\sigma$-finite Borel measure on $R$. Then extreme positive contractions and extreme contractions in $\mathcal{B}(L^1(\mu_1), L^1(\mu_2))$ are exposed. Moreover the exposed operators are strongly exposed if and only if $L^1(\mu_1)$ is finite-dimensional.

**Proof:** Let $T_0$ be an extreme operator in $\mathcal{B}(L^1(\mu_1), L^1(\mu_2))$. Then $T_0^*$ is also extreme, so exposed. Suppose that the functional $\xi_0$ defined on $\mathcal{B}(L^\infty(\mu_2), L^\infty(\mu_1))$ exposes $T_0^*$. We define a functional $\eta$ on $\mathcal{B}(L^1(\mu_1), L^1(\mu_2))$ by $\eta(T) = \xi_0(T^*)$. It is easy to see that $\eta$ exposes $T_0$.
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