Application of Weierstrass units to relative power integral bases

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Abstract. Let $K$ be an imaginary quadratic field not equal to $\mathbb{Q}(\sqrt{-1})$ or $\mathbb{Q}(\sqrt{-3})$. We construct relative power integral bases between certain abelian extensions of $K$ in terms of Weierstrass units.

1. Introduction

Let $L/F$ be an extension of number fields and let $\mathcal{O}_L$ and $\mathcal{O}_F$ be the rings of integers of $L$ and $F$, respectively. We say that an element $\alpha$ of $L$ forms a relative power integral basis for $L/F$ if $\mathcal{O}_L = \mathcal{O}_F[\alpha]$. For example, if $N$ is a positive integer, then $\zeta_N = e^{2\pi i/N}$ forms a (relative) power integral basis for the extension $\mathbb{Q}(\zeta_N)/\mathbb{Q}$ (see Theorem 2.6 in [21]). In general not much is known about relative power integral bases except for extensions of degree less than or equal to 9 (see references [1]–[12]).

Let $K$ be an imaginary quadratic field not equal to $\mathbb{Q}(\sqrt{-1})$ or $\mathbb{Q}(\sqrt{-3})$. Let $m$ and $n$ be positive integers such that $m$ has at least two prime factors and each prime factor of $mn$ splits in $K/\mathbb{Q}$. In this paper we shall show that a certain Weierstrass unit forms a relative power integral basis for the ray class field modulo $(mn)$ over the compositum of the ray class field modulo $(m)$ and the ring class field of the order of conductor $mn$ of $K$ (Theorem 4.1). To this end, we shall make use of an explicit description of the Shimura reciprocity law in [20] due to Stevenhagen.

2. Weierstrass units

For a positive integer $N$, let $\Gamma(N)$ be the principal congruence subgroup of level $N$, namely
\[ \Gamma(N) = \{ \gamma \in \text{SL}_2(\mathbb{Z}) \mid \gamma \equiv I_2 \pmod{N} \}. \]
Then $\overline{\Gamma(N)} = \langle \Gamma(N), -I_2 \rangle / \{ \pm I_2 \}$ acts on the complex upper half-plane $\mathbb{H}$ by fractional linear transformations.

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Lemma 2.1. Let \( \Lambda = \omega_1 + \omega_2 \mathbb{Z} \) be a lattice in \( \mathbb{C} \). The Weierstrass \( \wp \)-function relative to \( \Lambda \) is defined by
\[
\wp(z; \Lambda) = \frac{1}{z^2} + \sum_{\omega \in \Lambda - \{0\}} \left\{ \frac{1}{(z - \omega)^2} - \frac{1}{\omega^2} \right\} \quad (z \in \mathbb{C}).
\]
It is a meromorphic function on \( z \), periodic with respect to \( \Lambda \).

Lemma 2.2. Let \( z_1, z_2 \in \mathbb{C} - \Lambda \). Then, \( \wp(z_1; \Lambda) = \wp(z_2; \Lambda) \) if and only if \( z_1 \equiv z_2 \pmod{\Lambda} \).

Proof. See Section 3 of Chapter IV in [19].

Let \( \Gamma = (1/\mathbb{N})\mathbb{Z}^2 - \mathbb{Z}^2 \) for an integer \( N \geq 2 \). We define
\[
\wp(\tau; r\mathbb{Z}) = \wp(\tau + s; r; 1) \quad (\tau \in \mathbb{H}).
\]
This is a weakly holomorphic (that is, holomorphic on \( \mathbb{H} \)) modular form of level \( N \) and weight 2 (see Chapter 6 in [16]). We further define
\[
g_2(\tau) = 60 \sum_{\omega \in [\tau, 1] - \{0\}} \frac{1}{\omega^4}, \quad g_3(\tau) = 140 \sum_{\omega \in [\tau, 1] - \{0\}} \frac{1}{\omega^6}, \quad \Delta(\tau) = g_2(\tau)^3 - 27g_3(\tau)^2,
\]
which are modular forms of level 1 and weights 4, 6, and 12, respectively. Now we define the Fricke function \( f_{\Gamma}(\tau) \) by
\[
f_{\Gamma}(\tau) = \frac{g_2(\tau)g_3(\tau)}{\Delta(\tau)} \wp(\tau; \Gamma).
\]
(2.1)

It depends only on \( \pm \Gamma \pmod{\mathbb{Z}^2} \) (see p. 8 in [16]) and is weakly holomorphic because \( \Delta(\tau) \) does not vanish on \( \mathbb{H} \).

Lemma 2.2. \( f_{\Gamma}(\tau) \) belongs to \( \mathcal{F}_N \) and satisfies the transformation formula
\[
f_{\Gamma}(\tau) = f_{\Gamma}(\gamma \tau) \quad (\gamma \in \text{GL}_2(\mathbb{Z}/N\mathbb{Z})/\{\pm I_2\} \simeq \text{Gal}(\mathcal{F}_N/\mathcal{F}_1)),
\]
where \( \gamma^\top \) stands for the transpose of \( \gamma \).

Proof. See Sections 2 and 3 of Chapter 6 in [16].
Proposition 2.5. Consider integers \( \alpha, \beta \) and \( \gamma \), it belongs to \( \mathcal{O}_{\mathbb{Z}} \).

Lemma 2.3. Let \( h_r \) be the primitive denominator of \( [r] \) (that is, \( M \) is the least positive integer so that \( M r, Ms \in \mathbb{Z} \)).

(i) \( g_{[r]}(\tau)^{12M} \) and \( g_{[s]}(\tau) \) are modular units of levels \( M \) and \( 12M^2 \), respectively.

(ii) \( g_{[r]}(\tau)^{12M} \) depends only on \( \pm [r] \pmod{\mathbb{Z}^2} \) and satisfies the transformation formula

\[ (g_{[r]}(\tau))^{12M} = g_{[r]}(\tau)^{12M} (\gamma \in \text{GL}_2(\mathbb{Z}/M\mathbb{Z})/\{\pm 1\} \simeq \text{Gal}(\mathcal{F}M/\mathcal{F}_1)). \]

(iii) Moreover, if \( M \) has at least two prime factors, then \( g_{[r]}(\tau) \) is a modular unit over \( \mathbb{Z} \).

Proof. (i) See Theorem 1.2 in Chapter 2 and Theorems 5.2 and 5.3 in Chapter 3 of [15].

(ii) See Proposition 1.4 in Chapter 2 of [15].

(iii) See Theorem 2.2 (i) in Chapter 2 of [15]. \( \square \)

Lemma 2.4. Let \( \alpha, \beta \in \mathbb{Q}^2 - \mathbb{Z}^2 \) be such that \( \alpha \neq \beta \pmod{\mathbb{Z}^2} \). We have the relation

\[ \varphi_{[\alpha]}(\tau) - \varphi_{[\beta]}(\tau) = \frac{g_{[\alpha]}(\tau)g_{[\beta]}(\tau)\eta(\tau)^4}{g_{[\alpha]}(\tau)^2g_{[\beta]}(\tau)^2}, \]

where

\[ \eta(\tau) = \sqrt{2\pi}\zeta s q^{1/24} \prod_{n=1}^{\infty}(1 - q^n). \]

Proof. See page 51 of [15]. \( \square \)

Proposition 2.5. Consider integers \( m \geq 2 \) and \( n > 0 \). The function

\[ h_{m,n}(\tau) = \frac{\varphi_{[0]}(\tau) - \varphi_{[1/m]}(\tau)}{\varphi_{[0]}(\tau) - \varphi_{[1/m]}(\tau)} \]

is a modular unit of level \( mn \). If \( m \) has at least two prime factors, then \( h_{m,n}(\tau) \) is a modular unit over \( \mathbb{Z} \).

Proof. It follows from Lemma 2.1 that the denominator of \( h_{m,n}(\tau) \) is not the zero function. Furthermore, since

\[ h_{m,n}(\tau) = \frac{f_{[0]}(\tau) - f_{[1/m]}(\tau)}{f_{[0]}(\tau) - f_{[1/m]}(\tau)} \]

by Definition (2.1), it belongs to \( \mathcal{F}_{mn} \), by Lemma 2.2.
On the other hand, we see that

\[ h_{m,n}(\tau) = -g_{1/m, 1/mn}(\tau) \eta(\tau)^4 / g_{1/m, 0}(\tau)^2 \]

by Lemma 2.4. This yields, by Lemma 2.3(i), that \( h_{m,n}(\tau) \) is a modular unit. Moreover, if \( m \) has at least two prime factors, then each of

\[ g_{1/m, 1/mn}(\tau), g_{1/m, 0}(\tau), g_{0, 1/mn}(\tau) \]

has primitive denominator with at least two prime factors. Therefore \( h_{m,n}(\tau) \) is a modular unit over \( \mathbb{Z} \), by Lemma 2.3(iii).

**Remark 2.6.** The modular unit \( h_{m,n}(\tau) \) is called a Weierstrass unit (see Section 6 in Chapter 2 of [15]).

### 3. The Shimura reciprocity law

Throughout this section let \( K \) be an imaginary quadratic field of discriminant \( d_K \) not equal to \( \mathbb{Q}(\sqrt{-1}) \) or \( \mathbb{Q}(\sqrt{-3}) \), and set

\[ \theta_K = \frac{d_K + \sqrt{d_K}}{2}. \]

This belongs to \( \mathbb{H} \) and forms a (relative) power integral basis for \( K/\mathbb{Q} \). Furthermore, \( g_2(\theta_K) \) and \( g_3(\theta_K) \) are nonzero (see p. 37 in [16]).

For a nonzero ideal \( \mathfrak{f} \) of \( \mathcal{O}_K \) we denote the ray class field modulo \( \mathfrak{f} \) by \( K_{\mathfrak{f}} \). Furthermore, if \( \mathfrak{O} = [N\theta_K, 1] \) is the order of conductor \( N \geq 1 \) of \( K \), then we mean the ring class field of the order \( \mathfrak{O} \) by \( H_{\mathfrak{O}} \). As a consequence of the main theorem of complex multiplication we have the following lemma.

**Lemma 3.1.** Let \( N \) be a positive integer.

(i) We have \( K_{(\mathfrak{N})} = K(\mathfrak{f}(\theta_K) \mid \mathfrak{f} \in \mathcal{F}_N \) is finite at \( \theta_K \)).

(ii) If \( N \geq 2 \), then \( K_{(\mathfrak{N})} = K_{(1)}(f_{1/N}[\theta_K]). \)

**Proof.** (i) See the corollary to Theorem 2 in Chapter 10 of [16].

(ii) See the corollary to Theorem 7 in Chapter 10 of [16].
Lemma 3.2. If $\theta \in \mathbb{H}$ is imaginary quadratic, then $j(\theta)$ is an algebraic integer.

Proof. See Theorem 4.14 in [18].

Proposition 3.3. Consider integers $m \geq 2$ and $n > 0$. Then $h_{m,n}(\theta_K)$ generates $K_{(mn)}$ over $K_{(m)}$. Moreover, if $m$ has at least two prime factors, then $h_{m,n}(\theta_K)$ is a unit of $\mathcal{O}_{K_{(mn)}}$.

Proof. We first derive that

$$K_{(mn)} = K_{(1)}(f_{\frac{1}{1/mn}}(\theta_K)) = K_{(m)}\left(\frac{f_{\frac{1}{1/mn}}(\theta_K) - f_{\frac{1}{1/m}}(\theta_K)}{f_{\frac{1}{1/m}}(\theta_K) - f_{\frac{1}{1/mn}}(\theta_K)}\right) = K_{(m)}(h_{m,n}(\theta_K))$$

(by Lemma 3.1 (i))

If $m$ has at least two prime factors, then $h_{m,n}(\tau)$ is a modular unit over $\mathbb{Z}$ by Proposition 2.5; hence $h_{m,n}(\tau)$ and $h_{m,n}(\tau)^{-1}$ are both integral over $\mathbb{Z}[j(\tau)]$. Therefore we conclude by Lemma 3.2 that $h_{m,n}(\theta_K)$ is a unit as an algebraic integer.

Lemma 3.4 (Shimura reciprocity law). Let $N$ be a positive integer and let $\mathcal{O}$ be the order of conductor $N$ of $K$. Consider the matrix group

$$W_{K,N} = \left\{ \begin{bmatrix} t - B_K s & -C_K s \\ s & t \end{bmatrix} \in \text{GL}_2(\mathbb{Z}/N\mathbb{Z}) \mid t, s \in \mathbb{Z}/N\mathbb{Z} \right\},$$

where

$$\min(\theta_K, \mathbb{Q}) = X^2 + B_K X + C_K = X^2 - d_K X + \frac{d_K^2 - d_K}{4}.$$  

(i) The map

$$W_{K,N}/\{\pm I_2\} \rightarrow \text{Gal}(K_{(N)}/K_{(1)})$$

$$\alpha \mapsto (f(\theta_K) \mapsto f^\alpha(\theta_K) \mid f(\tau) \in \mathcal{F}_N \text{ is finite at } \theta_K)$$

is an isomorphism.

(ii) The map of (i) induces an isomorphism

$$\{tI_2 \in \text{GL}_2(\mathbb{Z}/N\mathbb{Z}) \mid t \in (\mathbb{Z}/N\mathbb{Z})^* \}/\{\pm I_2\} \rightarrow \text{Gal}(K_{(N)}/H_{\mathcal{O}}).$$

(iii) If $M$ is a divisor of $N$, then we get an isomorphism

$$\{tI_2 \in \text{GL}_2(\mathbb{Z}/N\mathbb{Z}) \mid t \in (\mathbb{Z}/N\mathbb{Z})^* \text{ with } t \equiv \pm 1 \pmod{M} \}/\{\pm I_2\} \rightarrow \text{Gal}(K_{(N)}/K_{(M)}/H_{\mathcal{O}}).$$

Proof. (i) See Section 3 in [20].

(ii) See Proposition 5.3 in [14].

(iii) This is a direct consequence of (i) and (ii).
Lemma 3.5. Let $N \geq 2$ be an integer for which $(N) = N\mathcal{O}_K$ is not a power of a prime ideal.

(i) $g_{[1/N]}^{\theta_K}\{12N\}$ is a unit of $\mathcal{O}_{K(N)}$.

(ii) If $u$ is an integer prime to $N$, then $g_{[u/N]}^{\theta_K}\{12N\}$ is also a unit of $\mathcal{O}_{K(N)}$.

Proof. (i) See Remark 4.3 in [13] and [17] (or p. 293 in [16]).

(ii) We obtain

$$g_{[u/N]}^{\theta_K}\{12N\} = g_{[1/N]}^{(uI_2)}(\theta_K)^{12N}$$

(by Lemma 2.3(i) and (ii))

$$= (g_{[1/N]}^{(r)^{12N}})^{(uI_2)}(\theta_K)$$

(by Lemmas 3.1(i) and 3.4(i)).

Now, the result follows from (i).

 Remark 3.6. The singular value $g_{[1/N]}^{\theta_K}\{12N\}$ is called a Siegel–Ramachandra invariant modulo $(N)$, and it forms a normal basis for $K(N)/K$ (see [13]).

4. Construction of relative power integral bases

We are ready to prove our main theorem concerning relative power integral bases.

Theorem 4.1. Let $K$ be an imaginary quadratic field not equal to $\mathbb{Q}(\sqrt{-1})$ or $\mathbb{Q}(\sqrt{-3})$. Consider integers $m \geq 2$ and $n > 0$ such that

(i) $m$ has at least two prime factors,

(ii) each prime factor of $mn$ splits in $K/\mathbb{Q}$.

If $L = K_{nn}$ and $F = K_{mH_0}$ with $\mathcal{O}$ the order of conductor $mn$ of $K$, then $h_{m,n}(\theta_K)$ forms a relative power integral basis for $L/F$.

Proof. Let $\alpha = h_{m,n}(\theta_K)$. Since $\alpha$ is a unit of $\mathcal{O}_L$ by Proposition 3.3, we have the inclusion

$$\mathcal{O}_L \supseteq \mathcal{O}_F[\alpha].$$

For the converse, let $\beta$ be an element of $\mathcal{O}_L$. Since $L = F(\alpha)$ by Proposition 3.3, we can express $\beta$ as

$$\beta = c_0 + c_1 \alpha + \cdots + c_{\ell-1} \alpha^{\ell-1}$$

for some $c_0, c_1, \ldots, c_{\ell-1} \in F$, where $\ell = [L : F]$. In order to prove the converse inclusion $\mathcal{O}_L \subseteq \mathcal{O}_F[\alpha]$ it suffices to show that $c_0, c_1, \ldots, c_{\ell-1} \in \mathcal{O}_F$. Multiplying both sides of (4.1) by $\alpha^k$ ($k = 0, 1, \ldots, \ell - 1$) yields

$$c_0 \alpha^k + c_1 \alpha^{k+1} + \cdots + c_{\ell-1} \alpha^{k+\ell-1} = \beta \alpha^k.$$
Now, we take the trace $\text{Tr} = \text{Tr}_{L/F}$ to obtain

$$c_0 \text{Tr}(\alpha^k) + c_1 \text{Tr}(\alpha^{k+1}) + \cdots + c_{\ell-1} \text{Tr}(\alpha^{k+\ell-1}) = \text{Tr}(\beta \alpha^k).$$

Then we obtain the linear system (in the unknowns $c_0, c_1, \ldots, c_{\ell-1}$)

$$T \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{\ell-1} \end{bmatrix} = \begin{bmatrix} \text{Tr}(\beta) & \text{Tr}(\alpha) & \cdots & \text{Tr}(\alpha^{\ell-1}) \\ \text{Tr}(\alpha) & \text{Tr}(\alpha^2) & \cdots & \text{Tr}(\alpha^\ell) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Tr}(\alpha^{\ell-1}) & \text{Tr}(\alpha^\ell) & \cdots & \text{Tr}(\alpha^{2\ell-2}) \end{bmatrix},$$

where $T = \begin{bmatrix} \text{Tr}(1) & \text{Tr}(\alpha) & \cdots & \text{Tr}(\alpha^{\ell-1}) \\ \text{Tr}(\alpha) & \text{Tr}(\alpha^2) & \cdots & \text{Tr}(\alpha^\ell) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Tr}(\alpha^{\ell-1}) & \text{Tr}(\alpha^\ell) & \cdots & \text{Tr}(\alpha^{2\ell-2}) \end{bmatrix}$.

Since $\alpha, \beta \in \mathcal{O}_L$, all the entries of $T$ and $\begin{bmatrix} \text{Tr}(\beta) & \text{Tr}(\alpha) & \cdots & \text{Tr}(\alpha^{\ell-1}) \end{bmatrix}$ lie in $\mathcal{O}_F$. Hence we get

$$c_0, c_1, \ldots, c_{\ell-1} \in \frac{1}{\det(T)} \mathcal{O}_F.$$

Let $\alpha_1, \alpha_2, \ldots, \alpha_\ell$ be the conjugates of $\alpha$ via $\text{Gal}(L/F)$. We then derive that

$$\det(T) = \left| \begin{array}{cccc} \sum_{k=1}^{\ell} \alpha_0^k & \sum_{k=1}^{\ell} \alpha_1^k & \cdots & \sum_{k=1}^{\ell} \alpha_\ell^{k-1} \\ \sum_{k=1}^{\ell} \alpha_0^k & \sum_{k=1}^{\ell} \alpha_1^k & \cdots & \sum_{k=1}^{\ell} \alpha_\ell^{k-1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^{\ell} \alpha_0^{\ell-k} & \sum_{k=1}^{\ell} \alpha_1^{\ell-k} & \cdots & \sum_{k=1}^{\ell} \alpha_\ell^{2\ell-2} \end{array} \right|$$

$$= \prod_{1 \leq k_1 < k_2 \leq \ell} (\alpha_{k_1} - \alpha_{k_2})^2 \quad (\text{by the Vandermonde determinant formula})$$

$$= \pm \prod_{\sigma_1 \neq \sigma_2 \in \text{Gal}(L/F)} (\alpha^{\sigma_1} - \alpha^{\sigma_2})$$

$$= \pm \prod_{\sigma_1 \neq \sigma_2 \in \text{Gal}(L/F)} (\alpha^{\sigma_1 \sigma_2^{-1}} - \alpha)^{\sigma_2}.$$ (4.2)

If $\sigma$ is a nonidentity element of $\text{Gal}(L/F)$, then by Lemma 3.4(iii) one can set $\sigma = tI_2$ for some $t \in \mathbb{N}$ such that

$$\gcd(t, mn) = 1, \quad t \equiv \pm 1 \pmod{m} \quad \text{and} \quad t \not\equiv \pm 1 \pmod{mn}.$$
Thus we deduce that

$$\alpha^\sigma - \alpha = h_{m,n}(\theta_K)^\sigma - h_{m,n}(\theta_K)$$

$$= \left( \frac{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)}{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)} \right) \sigma - \frac{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)}{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)}$$

(by (2.3))

$$= \frac{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)}{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)}$$

(by Lemmas 3.4(iii) and 2.2)

$$= \frac{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)}{f_{1/m}[0](\theta_K) - f_{1/m}[0](\theta_K)}$$

(by Definition (2.1))

$$= \frac{g_{[t/mn]}(\theta_K)g_{[-1/m]}(\theta_K)g_{[0]}(\theta_K)^2g_{[1/m]}(\theta_K)^2}{g_{[1/m]}(\theta_K)g_{[-1/m]}(\theta_K)g_{[0]}(\theta_K)^2g_{[1/m]}(\theta_K)^2}$$

(by Lemma 2.4).

Since each of

$$\left[0, \frac{1}{m} \right], \left[\frac{1}{m}, \frac{1}{m} \right], \left[\frac{1}{m}, \frac{1}{m} \right], \left[\frac{-1}{m}, \frac{1}{m} \right], \left[0, \frac{1}{m} \right], \left[0, \frac{1}{m} \right]$$

has by the hypothesis (i) primitive denominator with at least two prime factors, the values

$$g_{[0]}(\theta_K), g_{[1/m]}(\theta_K), g_{[1/m]}(\theta_K), g_{[-1/m]}(\theta_K), g_{[0]}(\theta_K), g_{[1/m]}(\theta_K)$$

are units as algebraic integers by Lemmas 2.3(iii) and 3.2. On the other hand, set

$$t + \frac{1}{m} = \frac{a}{N}$$

for some relatively prime positive integers $N$ and $a$.

Since $t \not\equiv \pm 1 \pmod{mn}$, we get $N \geq 2$. Moreover, $(N) = \mathcal{O}_K$ is not a power of a prime ideal by the hypothesis (ii). So $g_{[t+1/mn]}(\theta_K) = g_{[a/N]}(\theta_K)$ is a unit as an algebraic integer by Lemma 3.5(ii). In a similar fashion, we also see that $g_{[t-1/mn]}(\theta_K)$ is a unit as an algebraic integer. Therefore $\alpha^\sigma - \alpha$ is a unit of $\mathcal{O}_L$.

This implies that $\det(T)$ is a unit of $\mathcal{O}_F$ by (4.2), and hence we get the converse inclusion

$$\mathcal{O}_L \subseteq \mathcal{O}_F[\alpha]$$

as desired. \[\square\]

**Remark 4.2.** Since $\mathcal{O}_L = \mathcal{O}_F[\alpha]$ and the discriminant of $\alpha$ is a unit of $\mathcal{O}_F$, $L/F$ is an unramified extension.
References


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