Correction to 

**Products of Distributions: Nonstandard Methods**

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The purpose of this note is to correct a faulty structure theorem for limited distributions, claimed to hold in [1] in the setting of Internal Set Theory. Though the assertion could not be recovered completely, a weaker version suffices to prove those results in [1] which depended on it.

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I am grateful to T. D. TODOROV, who pointed out the following mistake in [1] to me: The sentence after equation (2.4) is wrong; it does not follow that \( \sup \{|g(x)| : |x| \leq k\} \) is infinitesimal. A counterexample is obtained by taking \( \theta \) as in (2.1) with \( \theta(0) = 1 \) and letting \( T(x) = e^{n/\rho} \theta(e^{1/\rho} x) \); then \( g(x) = T(x) - \rho^{-n} \theta(x/\rho) \) is not infinitesimal at \( x = 0 \). This invalidates the proof of (a) \( \Rightarrow \) (b) in Proposition 2.10. For the same reason, Corollary 2.11 is wrong; the same \( T(x) \) as above supplies a counterexample. The implication (b) \( \Rightarrow \) (a) in Proposition 2.10 remains correct. Corollary 2.11 is not a consequence of Proposition 2.10, but rather of its proof. Thus (a) \( \Rightarrow \) (b) may still be true, though I do not have a proof at the moment.

The only effect of the invalidation of Corollary 2.11 occurs in Section 4. To remedy this, Proposition 2.10 and Corollary 2.11 should be replaced by

**Proposition 2.12:** Let \( T \in ^*\mathcal{D}' \) and \( \rho \in \mathcal{R} \) be a positive infinitesimal, \( \theta \in ^*\mathcal{D} \) and \( \theta_\rho \) as in (2.1). Then

\[
\forall ^*k \in \mathcal{N} \exists ^*j \in \mathcal{N} \text{ such that } \sup \{|(T \ast \theta_\rho)(x)| : |x| \leq k\} \leq \rho^{-j}.
\]

**Proof:** By the classical structure theorem for distributions and transfer we have: Given \( k \in ^*\mathcal{N} \) there is \( \alpha \in ^*\mathcal{N}^\alpha_0 \) and a standard continuous function \( f \) with compact support such that \( \langle T, \varphi \rangle = (-1)^{|\alpha|} \langle f, \partial^\alpha \varphi \rangle \) for all \( \varphi \in \mathcal{D}_{k+1} \). Now if \( \text{supp}(\varphi) \subset \{x : |x| \leq k + \frac{1}{2}\} \), then \( \varphi \ast \theta_\rho \in \mathcal{D}_{k+1} \), so

\[
(T \ast \theta_\rho, \varphi) = \langle T, \varphi \ast \theta_\rho \rangle = (-1)^{|\alpha|} \langle f, \partial^\alpha (\varphi \ast \theta_\rho) \rangle = \langle \partial^\alpha (f \ast \theta_\rho), \varphi \rangle.
\]

It follows that \( (T \ast \theta_\rho)(x) = \partial^\alpha (f \ast \theta_\rho)(x) \) for all \( x \) with \( |x| \leq k \). The assertion that \( |\partial^\alpha (f \ast \theta_\rho)(x)| \leq \rho^{-|\alpha|-1} \) for all \( |x| \leq k \) is verified just as in the proof of Corollary 2.11.
Turning to Section 4, we infer from the discussion above that the first sentence after Definition 4.1 is incorrect: \( D' \) is not contained in \( E_p \). However, thanks to Proposition 2.12, it remains true that the map \( S \to S \ast \theta_p \) defines an imbedding of \( \ast D' \) into \( E_p \). Finally, Proposition 4.4 remains true, but a slight modification of its proof is required. To show that \( S \ast \theta_p \in E_p \) for \( S \in \ast S' \) and \( \theta \in \ast S \), a similar argument as in Proposition 2.12 may be used, but this time involving the structure theorem for tempered distributions.

I may take the opportunity to point out that two open questions in the article have been answered by now. R. WAWAK [2] has shown that the products \( M_2, M_3, M_4 \) actually are equivalent. In particular, the existence of either \( M_2, M_3, or M_4 \) implies the existence of \( M_5 \). On the other hand, J. JELÍNEK [2] has shown that \( M_5 \) is strictly more general than \( M_2 - M_4 \). I would also like to call the readers' attention to an interesting recent characterization of the products of type (P1) for homogeneous distributions by P. WAGNER [3].

REFERENCES


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