Existence, Uniqueness and Data Dependence for the Solutions of some Integro-Differential Equations of Mixed Type in Banach Space

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Abstract. In this paper we study existence, uniqueness and data dependence for the solutions of some integro-differential equations of mixed type in Banach space by using Picard and weakly Picard operators’ technique and suitable Bielecki norms.

Keywords: Integro-differential equations, fixed points, Picard operators, weakly Picard operators

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1. Introduction

Ordinary differential equations, functional differential equations with or without deviating argument and equations in abstract spaces have been studied in many papers. In the papers [3, 6] theorems about the existence and uniqueness of solutions of some abstract nonlinear non-local Cauchy problems in Banach spaces were considered and in the paper [4] a theorem about the existence of an approximate solution to an abstract nonlinear non-local Cauchy problem in a Banach space was given, too. We remark in the same field the monographs [5, 9, 11 - 13].

Integro-differential equations of mixed type in Banach spaces have been studied in the papers [7, 10], and integro-differential equations of mixed type with impulses in Banach spaces were considered in the paper [14], too. Fredholm-Volterra integral equations in relationship with Maia’s theorem were considered in the paper [16].

The aim of the present paper is to obtain existence, uniqueness and data dependence results for the solutions of some integro-differential equations of
mixed type in Banach space. To do this we use Picard and weakly Picard operators’ technique due to I. A. Rus (see [18 - 22]). So, our technique is different from those used in the papers quoted above.

Let \((X, \| \cdot \|)\) be a Banach space. Consider the problem

\[
x'(t) = f\left(t, x(t), \int_0^t K_1(t, s)x(s) \, ds, \int_0^T K_2(t, s)x(s) \, ds\right) \\
x(0) = x_0
\]

on \([0, T]\), where \(f \in C([0, T] \times X^3, X), K_i \in C(D_i, \mathbb{R}) \ (i = 1, 2)\) and \(x_0 \in X\).

It is well known that \(x \in C^1([0, T], X)\) is a solution of problem (1) if and only if \(x\) is a solution in \(C([0, T], X)\) of the integro-differential equation

\[
x(t) = x_0 + \int_0^t f\left(\xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s) \, ds, \int_0^T K_2(\xi, s)x(s) \, ds\right) \, d\xi
\]

on \([0, T]\).

In [10] the author combines topological degree theory and monotone iterative technique given in [12] to investigate the existence of solutions and also minimal and maximal solutions of problem (1). In the present paper we consider suitable Bielecki norms in a convenient space and obtain existence, uniqueness and data dependence results for the solutions of equation (2) which is equivalent to problem (1).

In [7] the authors study the existence of solutions of the abstract non-local integro-differential Cauchy problem in arbitrary Banach spaces

\[
x'(t) = f\left(t, x(t), \int_0^t K_1(t, s)x(s) \, ds, \int_0^T K_2(t, s)x(s) \, ds\right) \\
x(0) = x_0 - \sum_{i=1}^p c_i x(t_i)
\]

on \([0, T]\), where \(f \in C([0, T] \times X^3, X), 0 < t_1 < t_2 < \ldots < t_p \leq T, c_i \neq 0, p \in \mathbb{N}\) and \(x_0 \in X\). This problem is equivalent to the integro-differential equation

\[
x(t) = x_0 - \sum_{i=1}^p c_i x(t_i) \\
+ \int_0^t f\left(\xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s) \, ds, \int_0^T K_2(\xi, s)x(s) \, ds\right) \, d\xi
\]
on \([0, T]\). For this purpose, the Kuratowski measure of non-compactness, fixed point principles and a monotone iterative technique were applied. We remark that the weakly Picard operators technique can be used to prove existence of solutions to equation (3).

2. Preliminaries

Let \((X, d)\) be a metric space and \(A : X \rightarrow X\) an operator. We shall use the following notations:

\[
P(X) = \{ Y \subseteq X | Y \neq \emptyset \}
\]

\[
F_A = \{ x \in X | A(x) = x \} - \text{the fixed point set of } A
\]

\[
I(A) = \{ Y \in P(X) | A(Y) \subseteq Y \}
\]

\[
O_A(x) = \{ x, A(x), A^2(x), ..., A^n(x), ... \} - \text{the } A\text{-orbit of } x \in X
\]

\[
H : P(X) \times P(X) \rightarrow \mathbb{R}^+ \cup \{ +\infty \}
\]

\[
H(Y, Z) = \max \left( \sup_{a \in Y} \inf_{b \in Z} d(a, b), \sup_{b \in Z} \inf_{a \in Y} d(a, b) \right)
\] - the Pompeiu-Hausdorff functional on \(P(X)\).

**Definition 2.1** (Rus [18]). Let \((X, d)\) be a metric space. An operator \(A : X \rightarrow X\) is a Picard operator if there exists \(x^* \in X\) such that \(F_A = \{ x^* \}\) and the sequence \((A^n(x_0))_{n \in \mathbb{N}}\) converges to \(x^*\) for all \(x_0 \in X\).

**Definition 2.2** (Rus [19]). Let \((X, d)\) be a metric space. An operator \(A : X \rightarrow X\) is a weakly Picard operator if the sequence \((A^n(x_0))_{n \in \mathbb{N}}\) converges for all \(x_0 \in X\) and its limit (which may depend on \(x_0\)) is a fixed point of \(A\).

If \(A\) is a weakly Picard operator, then we consider the operator

\[
A^\infty : X \rightarrow X, \quad A^\infty(x) = \lim_{n \to \infty} A^n(x).
\]

The following results are useful in what follows:

**Theorem 2.1** [17]. Let \((Y, d)\) be a complete metric space and \(A, B : Y \rightarrow Y\) two operators. We suppose the following:

(i) \(A\) is a contraction with contraction constant \(\alpha\) and \(F_A = \{ x^*_A \}\).

(ii) \(B\) has fixed points and \(x^*_B \in F_B\).

(iii) There exists \(\eta > 0\) such that \(d(A(x), B(x)) \leq \eta\), for all \(x \in Y\).

Then \(d(x^*_A, x^*_B) \leq \frac{\eta}{1-\alpha}\).

**Theorem 2.2** [22]. Let \((X, d)\) be a complete metric space and \(A, B : X \rightarrow X\) two orbitally continuous operators. We suppose the following:
(i) There exists $\alpha \in [0, 1)$ such that
\[
  d(A^2(x), A(x)) \leq \alpha d(x, A(x)) \quad (x \in X).
\]
\[
  d(B^2(x), B(x)) \leq \alpha d(x, B(x)) \quad (x \in X).
\]

(ii) There exists $\eta > 0$ such that $d(A(x), B(x)) \leq \eta$ for all $x \in X$.

Then $H(F_A, F_B) \leq \frac{\eta}{1-\alpha}$ where $H$ denotes the Pompeiu-Hausdorff functional.

Theorem 2.3 [19]. Let $(X, d)$ be a metric space. Then $A : X \to X$ is a weakly Picard operator if and only if there exists a partition $X = \bigcup_{\lambda \in \Lambda} X_\lambda$ of $X$ such that

(a) $X_\lambda \in I(A)$

(b) $A|_{X_\lambda} : X_\lambda \to X_\lambda$ is a Picard operator, for all $\lambda \in \Lambda$.

Consider a Banach space $(X, \| \cdot \|)$, let $\| \cdot \|_B$ and $\| \cdot \|_C$ be the Bielecki and Chebyshev norms on $C([0, T], X)$ defined by
\[
  \|x\|_B = \max_{t \in [0, T]} \|x(t)e^{-\tau t}\| \quad (\tau > 0) \quad \text{and} \quad \|x\|_C = \max_{t \in [0, T]} \|x(t)\|
\]

and denote by $d_B$ and $d_C$ their corresponding metrics. We consider the set
\[
  C_L([0, T], X) = \left\{ x \in C([0, T], X) \left| \|x(t_1) - x(t_2)\| \leq L|t_1 - t_2| \right. \right. \text{for all } t_1, t_2 \in [0, T] \}
\]
where $L > 0$ and $B_R = \{ x \in X : \|x\| \leq R \}$ with $R > 0$. If $d \in \{d_C, d_B\}$, then $(C([0, T], X), d)$ and $(C_L([0, T], X), d)$ are complete metric spaces.

3. A integro-differential equation of mixed type

Consider equation (2). Denote $k_i = \max_{(t, s) \in D_i} |K_i(t, s)|$ ($i = 1, 2$). We have

Theorem 3.1. Suppose the following:

(i) $f \in C([0, T] \times X^3, X)$.

(ii) There exists a constant $M > 0$ such that $\|f(s, u, v, w)\| \leq M$ for all $u, v, w \in X$ and all $s \in [0, T]$.

(iii) $M \leq L$.

(iv) There exists a constant $L_0 > 0$ such that
\[
  \|f(s, u_1, v_1, w_1) - f(s, u_2, v_2, w_2)\|
  \leq L_0 \left( \|u_1 - u_2\| + \|v_1 - v_2\| + \|w_1 - w_2\| \right)
\]
for all \( u_i, v_i, w_i \in X \quad (i = 1, 2) \) and all \( s \in [0, T] \).

(v) There exists a constant \( \tau > 0 \) such that \( \frac{L_0}{\tau} \left( 1 + \frac{k_1}{\tau} + k_2 T e^{\tau T} \right) < 1 \).

Then equation (2) has a unique solution \( x^* \) in \( C_L([0, T], X) \), and this solution can be obtained by the successive approximation method, starting from any element of \( C_L([0, T], X) \).

**Proof.** Consider the continuous operator

\[
A : (C_L([0, T], X), \| \cdot \|_B) \rightarrow (C_L([0, T], X), \| \cdot \|_B)
\]

defined by

\[
A(x)(t) = x_0 + \int_0^t f \left( \xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s) \, ds, \int_0^T K_2(\xi, s)x(s) \, ds \right) d\xi.
\]

We have

\[
\| A(x)(t) - A(z)(t) \|
\leq \int_0^t \left\| f \left( \xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s) \, ds, \int_0^T K_2(\xi, s)x(s) \, ds \right) 
- f \left( \xi, z(\xi), \int_0^\xi K_1(\xi, s)z(s) \, ds, \int_0^T K_2(\xi, s)z(s) \, ds \right) \right\| d\xi 
\leq L_0 \int_0^t \left[ \| x(\xi) - z(\xi) \| + \left\| \int_0^\xi K_1(\xi, s)(x(s) - z(s)) \, ds \right\| 
+ \left\| \int_0^T K_2(\xi, s)(x(s) - z(s)) \, ds \right\| \right] d\xi 
\leq L_0 \left\{ \int_0^t \| x(\xi) - z(\xi) \| d\xi + k_1 \int_0^t \left( \int_0^\xi \| x(s) - z(s) \| \, ds \right) d\xi 
+ k_2 \int_0^t \left( \int_0^T \| x(s) - z(s) \| e^{-\tau s} e^{\tau s} \right) d\xi \right\} 
\leq L_0 \left\{ \int_0^t \| x(\xi) - z(\xi) \| e^{-\tau \xi} e^{\tau \xi} d\xi 
+ k_1 \int_0^t \left( \int_0^\xi \| x(s) - z(s) \| e^{-\tau s} e^{\tau s} ds \right) d\xi 
+ k_2 \int_0^t \left( \int_0^T \| x(s) - z(s) \| e^{-\tau s} e^{\tau s} ds \right) d\xi \right\}
\]
\[ \begin{align*}
&\leq L_0 \|x - z\|_B \left[ \int_0^t e^{\tau \xi} d\xi + k_1 \int_0^t \left( \int_0^\xi e^{\tau s} ds \right) d\xi \\
&\quad + k_2 \int_0^t \left( \int_0^T e^{\tau s} ds \right) d\xi \right] \\
&= L_0 \|x - z\|_B \left[ \left( \frac{e^{\tau t}}{\tau} - \frac{1}{\tau} \right) + k_1 \left( \frac{e^{\tau t}}{\tau} - \frac{1}{\tau} - t \right) + \frac{k_2}{\tau} (e^{\tau T} - 1)t \right] \\
&\leq L_0 \|x - z\|_B \left[ \frac{e^{\tau t}}{\tau} + \frac{k_1}{\tau} \frac{e^{\tau t}}{\tau} + k_2 \frac{e^{\tau T}}{\tau} e^{\tau(T-t)T} \right] \\
&\leq L_0 \frac{1}{\tau} e^{\tau t} \left( 1 + \frac{k_1}{\tau} + k_2 Te^{\tau T} \right) \|x - z\|_B
\end{align*} \]

for all \( x, z \in C_L([0, T], X) \). It follows that

\[ \|A(x)(t) - A(z)(t)\| e^{-\tau t} \leq \frac{L_0}{\tau} \left( 1 + \frac{k_1}{\tau} + k_2 Te^{\tau T} \right) \|x - z\|_B \]

for all \( t \in [0, T] \). So

\[ \|A(x) - A(z)\|_B \leq \frac{L_0}{\tau} \left( 1 + \frac{k_1}{\tau} + k_2 Te^{\tau T} \right) \|x - z\|_B \]

for all \( x, z \in C_L([0, T], X) \). The operator \( A \) is of Lipschitz type with constant

\[ L_A = \frac{L_0 \left( 1 + \frac{k_1}{\tau} + k_2 Te^{\tau T} \right)}{\tau} \tag{4} \]

and \( 0 < L_A < 1 \). By applying the Contraction Principle to this operator we obtain that \( A \) is a Picard operator. 

Similarly as above, we can prove

**Theorem 3.2.** Suppose the following:

(i) \( f \in C([0, T] \times B^3_R, X) \) with \( \|f(s, u, v, w)\| \leq M(R) \) for all \( s \in [0, T] \)
and \( u, v, w \in B_R \).

(ii) \( M(R) \leq L \).

(iii) \( k_i T \leq 1 \) \( (i = 1, 2) \).

(iv) \( \|x_0\| + M(R)T \leq R \).
(v) There exists a constant $L_0 > 0$ such that
\[
\| f(s, u_1, v_1, w_1) - f(s, u_2, v_2, w_2) \| \\
\leq L_0 (\| u_1 - u_2 \| + \| v_1 - v_2 \| + \| w_1 - w_2 \|)
\]
for all $u_i, v_i, w_i \in B_R \ (i = 1, 2)$ and all $s \in [0, T]$.

(vi) There exists a constant $\tau > 0$ such that
\[
\frac{L_0}{\tau} (1 + \frac{k_1}{\tau} + k_2 Te^{\tau T}) < 1.
\]

Then equation (2) has a unique solution in $C_L([0, T], B_R)$, and this solution can be obtained by the successive approximation method, starting from any element of $C_L([0, T], B_R)$.

Remark 3.1. If we consider the problem
\[
x' = \frac{1}{10} \int_0^t \sin(t + s)x(s) \, ds + \frac{1}{18} \int_0^t \cos(ts)x(s) \, ds
\]
\[
x(0) = 0
\]
on $[0, T]$, then $L_0 = 1$, $k_1 = \frac{1}{10}$, $k_2 = \frac{1}{18}$, and for $\tau = 2$ we have condition (vi) in Theorem 3.2.

Now, we consider both equation (2) and
\[
x(t) = y_0 + \int_0^t g \left( \xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s) \, ds, \int_0^T K_2(\xi, s)x(s) \, ds \right) d\xi
\]
on $[0, T]$, where $g \in C([0, T] \times X^3, X)$ and $K_i \in C(D_i, \mathbb{R}) \ (i = 1, 2)$ are the same as in equation (2) and $y_0 \in X$. We have

Theorem 3.3. Suppose the following:

(i) All conditions in Theorem 3.1 are satisfied and $x^* \in C_L([0, T], X)$ is the unique solution of equation (2).

(ii) There exists a constant $M_1 > 0$ such that $\| g(s, u, v, w) \| \leq M_1$ for all $u, v, w \in X$ and all $s \in [0, T]$.

(iii) With the same Lipschitz constant $L_0$ as in Theorem 3.1,
\[
\| g(s, u_1, v_1, w_1) - g(s, u_2, v_2, w_2) \| \\
\leq L_0 (\| u_1 - u_2 \| + \| v_1 - v_2 \| + \| w_1 - w_2 \|)
\]
for all $u_i, v_i, w_i \in X \ (i = 1, 2)$ and all $s \in [0, T]$.

(iv) $M_1 \leq L$.

(v) There exists a constant $\eta > 0$ such that
\[
\| f(s, u, v, w) - g(s, u, v, w) \| \leq \eta
\]
for all $u, v, w \in X$ and $s \in [0, T]$.

Then, if $y^*$ is the solution of equation (5),

$$\|x^* - y^*\|_B \leq \frac{\|x_0 - y_0\| + \eta T}{1 - L_A}$$

where $L_A$ is given by (4) with $\tau = \tau_0 > 0$ such that $0 < L_A < 1$.

**Proof.** Consider the operators

$$A, B : C_L([0, T], X) \to C_L([0, T], X)$$

defined by

$$A(x)(t) = x_0 + \int_0^t f(t, x(t), \int_0^t K_1(s, x(s)) ds, \int_0^T K_2(s, x(s)) ds) \, ds$$

$$B(x)(t) = y_0 + \int_0^t g(t, x(t), \int_0^t K_1(s, x(s)) ds, \int_0^T K_2(s, x(s)) ds) \, ds$$

on $[0, T]$, in which $K_i \in C(D_i, \mathbb{R})$ ($i = 1, 2$) are the same. We have

$$\|A(x)(t) - B(x)(t)\| \leq \|x_0 - y_0\| + \eta T \quad (t \in [0, T]).$$

It follows that

$$\|A(x) - B(x)\|_B \leq \|x_0 - y_0\| + \eta T.$$}

So we can apply Theorem 2.1.

**Remark 3.2.** The results obtained in this section can be generalized to study existence, uniqueness and data dependence for the solutions of the problem with linear modification of the argument

$$x'(t) = f(t, x(t), x(\lambda t), \int_0^t K_1(s, x(\lambda s)) ds, \int_0^T K_2(s, x(\lambda s)) ds)$$

$$x(0) = x_0$$

on $[0, T]$, where $0 < \lambda < 1$, $f \in C([0, T] \times X^4, X)$, $K_i \in C(D_i, R)$ ($i = 1, 2$) and $x_0 \in X$. This problem is more general than those considered in [15].
4. Another integro-differential equation of mixed type

Now, we consider the integral equation of mixed type

\[ x(t) = x(0) + \int_0^t f(\xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s)\,ds, \int_0^T K_2(\xi, s)x(s)\,ds)\,d\xi \]  

(6)
on \[0, T\], where \( f \in C([0, T] \times X^3, X) \), \( K_i \in C(D_i, \mathbb{R}) \) and \( D_i \) \((i = 1, 2)\) are as in problem (1). We have

**Theorem 4.1.** Suppose that for equation (6) the same conditions as in Theorem 3.1 are satisfied. Then this equation has solutions in \( C_L([0, T], X) \). If \( S \subset C_L([0, T], X) \) is its solutions set, then \( \text{card} \, S = \text{card} \, X \).

**Proof.** Consider the operator

\[ A_\ast : C_L([0, T], X) \to C_L([0, T], X) \]

defined by

\[ A_\ast(x)(t) = x(0) + \int_0^t f(\xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s)\,ds, \int_0^T K_2(\xi, s)x(s)\,ds)\,d\xi. \]

This is a continuous operator, but not a Lipschitz one. We can write

\[ C_L([0, T], X) = \bigcup_{\alpha \in X} X_\alpha, \quad X_\alpha = \{x \in C_L([0, T], X) : x(0) = \alpha\}. \]

We have that \( X_\alpha \) is an invariant set of \( A_\ast \) and we apply Theorem 3.1 to \( A_\ast|_{X_\alpha} \). By using Theorem 2.3 we obtain that \( A_\ast \) is a weakly Picard operator. Consider the operator

\[ A_\ast^\infty : C_L([0, T], X) \to C_L([0, T], X), \quad A_\ast^\infty(x) = \lim_{n \to \infty} A_\ast^n(x). \]

From \( A_\ast^{n+1}(x) = A_\ast(A_\ast^n(x)) \) and the continuity of \( A_\ast \), \( A_\ast^\infty(x) \in F_{A_\ast} \). Then \( A_\ast^\infty(C_L([0, T], X)) = F_{A_\ast} = S \), and \( S \neq \emptyset \). So, \( \text{card} \, S = \text{card} \, X \).

**Remark 4.1.** Similarly as above we can prove the existence of solutions of equation (3) that corresponds to a problem considered in [7].

In order to study data dependence for the solutions set of equation (6) we consider both (6) and the equation

\[ x(t) = x(0) + \int_0^t g(\xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s)\,ds, \int_0^T K_2(\xi, s)x(s)\,ds)\,d\xi \]
on \[0, T\] where \( K_1, K_2 \) are the same as in (6) and \( g \in C([0, T] \times X^3, X) \). Let \( S_1 \) be the solutions set of this equation.
Theorem 4.2. Suppose the following:

(i) There exists a constant $L_* > 0$ such that

\[
\|f(s, u_1, v_1, w_1) - f(s, u_2, v_2, w_2)\| \\
\leq L_* (\|u_1 - u_2\| + \|v_1 - v_2\| + \|w_1 - w_2\|) \\
\|g(s, u_1, v_1, w_1) - g(s, u_2, v_2, w_2)\| \\
\leq L_* (\|u_1 - u_2\| + \|v_1 - v_2\| + \|w_1 - w_2\|)
\]

for all $u_i, v_i, w_i \in X$ $(i = 1, 2)$ and all $s \in [0, T]$.

(ii) There exists a constant $M_* > 0$ such that

\[
\|f(s, u, v, w)\| \leq M_* \\
\|g(s, u, v, w)\| \leq M_*
\]

for all $u, v, w \in X$ and all $s \in [0, T]$.

(iii) $M_* \leq L_*$. 

(iv) There exists a constant $\eta_1 > 0$ such that

\[
\|f(s, u, v, w) - g(s, u, v, w)\| \leq \eta_1
\]

for all $u, v, w \in X$ and all $s \in [0, T]$.

(v) $3L_* T k_0 < 1$, where $k_0 = \max(1, k_1 T, k_2 T)$.

Then

\[
H_{\|\cdot\|_C}(S, S_1) \leq \frac{\eta_1 T}{1 - 3L_* T k_0}
\]

where by $H_{\|\cdot\|_C}$ we denote the Pompeiu-Hausdorff functional with respect to $\| \cdot \|_C$ on $C_L([0, T], X)$.

**Proof.** Consider the operator

\[
B_*: C_L([0, T], X) \rightarrow C_L([0, T], X)
\]

defined by

\[
B_*(x)(t) = x(0) + \int_0^t g(\xi, x(\xi), \int_0^\xi K_1(\xi, s)x(s) ds, \int_0^T K_2(\xi, s)x(s) ds) d\xi
\]

on $[0, T]$. We have

\[
\|A_*^2(x)(t) - A_*(x)(t)\| \\
\leq L_* \int_0^t \|A_*(x)(\xi) - x(\xi)\|
\]
\[
+ \left\| \int_0^\xi K_1(\xi, s)(A_*(x)(s) - x(s)) \, ds \right\| \\
+ \left\| \int_0^T K_2(\xi, s)(A_*(x)(s) - x(s)) \, ds \right\| \, d\xi \\
\leq 3L_*T \max(1, k_1T, k_2T) \|A_*(x) - x\|_C \\
= 3L_*Tk_0 \|A_*(x) - x\|_C
\]
for all \( x \in C_L([0, T], X) \). Similarly,
\[
\|B_2^2(x)(t) - B_2(x)(t)\| \leq 3L_*Tk_0 \|B_2(x) - x\|_C
\]
for all \( x \in C_L([0, T], X) \). It follows that
\[
\|A_2^2(x) - A_2(x)\|_C \leq 3L_*Tk_0 \|A_2(x) - x\|_C \\
\|B_2^2(x) - B_2(x)\|_C \leq 3L_*Tk_0 \|B_2(x) - x\|_C.
\]
Because of assumption (iv), \( \|A_*(x) - B_*(x)\|_C \leq \eta_1T \) for all \( x \in C_L([0, T], X) \).
By applying Theorem 2.2 we obtain \( H_{\|\|_C}(F_{A_*}, F_{B_*}) \leq \frac{\eta_1T}{1-3L_*Tk_0} \) and the theorem is proved.

References


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