Abstract

The question is studied whether weak solutions of linear partial integro-differential equations approach a constant spatial profile after rescaling, as time goes to infinity. The possible limits and corresponding scaling functions are identified and are shown to actually occur. The limiting equations are fractional diffusion equations which are known to have self-similar fundamental solutions. For an important special case, it is shown that the asymptotic profile is Gaussian and convergence holds in $L^2$, that is, solutions behave like fundamental solutions of the heat equation to leading order. Systems of integro-differential equations occurring in viscoelasticity are also discussed, and their solutions are shown to behave like fundamental solutions of a related Stokes system. The main assumption is that the integral kernel in the equation is regularly varying in the sense of Karamata.