New Trends in Teichmüller Theory and Mapping Class Groups

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Abstract. In this workshop, various topics in Teichmüller theory and mapping class groups were discussed. Twenty-three talks dealing with classical topics and new directions in this field were given. A problem session was organised on Thursday, and we compiled in this report the problems posed there.

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Introduction by the Organisers

The workshop New Trends in Teichmüller Theory and Mapping Class Groups, organised by Ken’ichi Ohshika (Osaka), Athanase Papadopoulos (Strasbourg), Robert Penner (Bures-sur-Yvette) and Anna Wienhard (Heidelberg) was attended by 50 participants, including a number of young researchers, with broad geographic representation from Europe, Asia and the USA. During the five days of the workshop, 23 talks were given, and on Thursday evening, a problem session was organised.

Teichmüller theory originates in the work of Teichmüller on quasi-conformal maps in the 1930s, and the study of mapping class groups was started by Dehn and Nielsen in the 1920s. The subjects are closely interrelated, since the mapping class group is the automorphism group of Teichmüller space with respect to its canonical complex structure and with respect to various metrics. Classically, Teichmüller...
theory is the study of moduli for complex structures on surfaces, but the expression has now a broader sense as the study of geometric structures on surfaces with several applications, and the various aspects of the theory are at the intersection of the fields of low-dimensional topology, algebraic topology, hyperbolic geometry, representations of discrete groups in Lie groups, symplectic geometry, topological quantum field theory, string theory, mathematical physics and others. All these interactions originate from the fact that Teichmüller space can be seen from various angles: as a space of equivalence classes of marked hyperbolic metrics, as a space of equivalence classes of complex algebraic curves, as a space of equivalence classes of marked conformal structures, as a space of equivalence classes of representations of the fundamental group of a surface into a Lie group \( \text{SL}(2, \mathbb{R}) \) and as a component of the moduli space of flat \( G \)-connections on a fixed surface where \( G \) is the Möbius group.

Since the works of Thurston starting in the 1970s, Teichmüller theory has a great impact on on low-dimensional geometry and topology. It has absorbed new techniques and viewpoints coming from complex analysis, combinatorial group theory, low-dimensional topology among others. Recently Teichmüller theory became a wider theory through its ramification and development into higher Teichmüller theory which extends this theory to representations of the surface into appropriate Lie groups \( G \). There is also a well-developed quantum Teichmüller theory, and a super Teichmüller theory which is beginning to grow. In fact, there was a substantive breakthrough during the workshop which produced the \( N=1 \) super McShane identity, the super version of an a priori constraint on length spectra that has found wide application in the classical theory. This super version has been long-sought in the physics community. The study of mapping class group is now closely related with thriving field of geometric group theory. All these developments were represented among the participants of the workshop.

The talks given in this workshop cover at the same time the new trends and the classical topics in a well balanced way. A relatively large amount of time for discussion was left.

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