

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Variational Methods for the Modelling of Inelastic Solids

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ABSTRACT. This workshop brought together two communities working on the same topic from different perspectives. It strengthened the exchange of ideas between experts from both mathematics and mechanics working on a wide range of questions related to the understanding and the prediction of processes in solids. Common tools in the analysis include the development of models within the broad framework of continuum mechanics, calculus of variations, nonlinear partial differential equations, nonlinear functional analysis, Gamma convergence, dimension reduction, homogenization, discretization methods and numerical simulations. The applications of these theories include but are not limited to nonlinear models in plasticity, microscopic theories at different scales, the role of pattern forming processes, effective theories, and effects in singular structures like blisters or folding patterns in thin sheets, passage from atomistic or discrete models to continuum models, interaction of scales and passage from the consideration of one specific time step to the continuous evolution of the system, including the evolution of appropriate measures of the internal structure of the system.

Mathematics Subject Classification (2010): 74Cxx, 74Bxx, 74Dxx, 74Rxx.

Introduction by the Organisers

The workshop *Variational Methods for the Modelling of Inelastic Solids*, organized by Georg Dolzmann (Regensburg), Adriana Garroni (Roma), Klaus Hackl (Bochum) and Michael Ortiz (Pasadena/Bonn) was well attended with over 50 participants with broad geographic representation including Austria, Germany, France, Italy, the Netherlands, Switzerland and the United States. The workshop

featured 23 presentations (eight presented by female participants, nine from the engineering community) and ample time for scientific discussions.

The presentations by Svendsen and Reese offered a detailed discussion of modeling in continuum mechanics, the lecture by Müller presented a completely new approach to elasticity based on data driven models and Ariza discussed models for hydrogen transport and storage, a crucial topic for the design of fuel cells. Effective behaviour and homogenization was featured by Francfort and Reina and topics related to microstructures and upscaling by Govindjee, Kochmann and Růland. Results in the framework of plasticity were presented by Luckhaus, Mora, Schweizer, and Weinberg and for the formulation and the analysis of fracture by Negri and Truskinovsky, while the challenging area of evolution was addressed by Davoli, Friedrich and Mielke. Finally stochastic aspects were introduced by Dal Maso. Special emphasis was given to the junior attendants including the Oberwolfach Leibniz Graduate Students (OLGS) and therefore one afternoon was dedicated to the presentations by Kreutz (spin systems), Frenzel (slip-stick motion), Waimann (cyclic loading), all of them OLGSs, and Ginster (strain gradient plasticity).

All participants were unanimous in their assessment that this meeting offered the most stimulating platform for the interaction between mathematics and mechanics in this field of outstanding importance in recent years. Many discussions will be continued and deepened within the departments of the participants and it is expected that long-lasting collaborations will emerge.

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