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Variational Methods for Evolution

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Abstract. Many evolutionary systems, as for example gradient flows or Hamiltonian systems, can be formulated in terms of variational principles or can be approximated using time-incremental minimization. Hence they can be studied using the mathematical techniques of the field of calculus of variations. This viewpoint has led to many discoveries and rapid expansion of the field over the last two decades. Relevant applications arise in mechanics of fluids and solids, in reaction-diffusion systems, in biology, in many-particle models, as well as in emerging uses in data science.

This workshop brought together a broad spectrum of researchers from calculus of variations, partial differential equations, metric geometry, and stochastics, as well as applied and computational scientists to discuss and exchange ideas. It focused on variational tools such as minimizing movement schemes, Gamma convergence, optimal transport, gradient flows, and large-deviation principles for time-continuous Markov processes.

Mathematics Subject Classification (2010): 49-06 (Calc. of Var.), 35-05 (PDEs),70-06 (mechanics of particles and systems), 58E30 (Variational principles), 60F10 (Large deviations), 82C05 (Classical dynamic and nonequilibrium statistical mechanics (general)).

Introduction by the Organisers

Variational approaches to evolution systems provide a rich and very active field of mathematical research combining several previously only loosely connected branches of mathematics. This includes studies of geometric aspects like metric structures in infinite dimensional function spaces and of the geometry of relevant energy landscapes, like their geodesic lambda-convexity. The connection with theory of optimal transport provides new insights into metric spaces on the one hand

and provides new tools for the theory of partial differential equations on the other hand. Another surprising connection links analysis and stochastics to provide new gradient structures for macroscopic partial differential equations arising from many-particle systems via large-deviation principles.

During the last few decades many evolutionary PDEs for models in mechanics, physics, chemistry, and biology have been studied via new approaches based on new variational techniques, combining well established tools and new ideas in clever ways. This includes classical and generalized gradient flows including rate-independent flows, geometric analysis for fluid and transport dynamics, Hamiltonian systems, and stochastic dynamics of many-particle systems. Concrete examples include the Fokker–Planck equation, porous medium equations, microfluidic systems and thin-film equations, interface evolutions, pattern formation and evolution, coarsening, micromagnetics, superconductors, materials science (crack propagation, behavior of material defects, epitaxial growth, grain boundary evolution), biological aggregation, many particle systems with interactions and randomness, and geometric flows.

The aim of this workshop was to bring together a group of experts and young researchers from calculus of variations, partial differential equations, non-smooth geometry, stochastic analysis, as well as applied and computational scientists for a stimulating interchange of ideas. This has succeeded remarkably well, and among the themes presented during the workshop, we mention here:

- optimal transport techniques and transportation distances, functional inequalities, entropic interpolation (many speakers);
- connections between gradient flows and other solution concepts, such as Brakke flow for mean curvature (Otto) and conservation laws (Brenier);
- connections between gradient flows, statistical mechanics, large deviations, and thermodynamics (Öttinger, Léonard, Maes, Zimmer);
- novel numerical methods for gradient flows and other variational evolutionary systems (Matthes, Pattacchini, Knees);
- new variational formulations (Stefanelli, Mittnenzweig, Dal Maso, Erbar, Monsaingeon), and new analysis of existing formulations (Savaré);
- reaction-diffusion problems as variational evolution (Liero);
- application of variational concepts for data analysis (García Trillos);
- entropy-entropy dissipation methods (Jüngel);
- crystallization, oscillation, and pattern formation (Niethammer, Maes);
- discrete interaction systems, evolution on graphs, and their metric and variational interpretation (Mittnenzweig, Niethammer, Erbar);
- rate-independent systems, quasi-static crack growth, and elasto-plasticity (Chambolle, Dal Maso);
- front propagation and phase-field models (Cancés, Otto).

In total, there were 17 talks of 45 minutes and 8 talks of 30 minutes leaving plenty of time for discussions, which have been greatly stimulated by the diversity of the topics and of the contributions. As always, the friendly atmosphere and the perfect environment of Oberwolfach have also contributed to the success of the meeting in a major way.

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Workshop: Variational Methods for Evolution

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