Abstract. In recent years a number of challenging research problems have crystallized in the analytic theory of Dirichlet series and its interaction with function theory in polydiscs. Their solutions appear to require unconventional combinations of expertise from harmonic, functional, and complex analysis, and especially from analytic number theory. This MFO workshop provided an ideal arena for the exchange of ideas needed to nurture further progress and to solve important problems.

Mathematics Subject Classification (2010): 11xx, 30xx.

Introduction by the Organisers

The workshop 'Interplay between number theory and analysis for Dirichlet series’, organised by Frédéric Bayart (Clermont Université), Kaisa Matomäki (University of Turku), Eero Saksman (University of Helsinki) and Kristian Seip (NTNU, Trondheim) was held October 29th – November 4th, 2017. This meeting was well attended with around 25 participants coming from a number of different countries, including participants from North America. The group formed a nice blend of researchers with somewhat different mathematical backgrounds which resulted in a fruitful interaction.

About 17 talks, of varying lengths, were delivered during the five days. The talks were given by both leading experts in the field as well as by rising young researchers. Given lectures dealt with e.g. connections between number theory and
random matrix theory, operators acting on Dirichlet series, distribution of Beurling primes, growth of $L^p$-norms of Dirichlet polynomials, growth and density of values of the Riemann zeta on boundary of the critical strip, Rado’s criterion for $k$th powers, Sarnak’s and Elliot’s conjectures, and Hardy type spaces of general Dirichlet series. In addition, a problem session was held on Wednesday evening. Two of the speakers gave wider expositions, each comprising of two talks: Hugh Montgomery’s beautiful review of the theory of mean values of Dirichlet polynomials was much appreciated by the participants. In turn, Adam Harper exposed his impressive solution of Helson’s conjecture, which has interesting consequences both to the analytic theory of Dirichlet series and to the growth of random or Dirichlet character sums. It also provided a surprising connection to the theory of multiplicative chaos.

The meeting stimulated many new collaborative research projects. Besides the high level scientific program, most of the participants took part in the classical social activity, i.e. the Wednesday afternoon hike to St. Roman for coffee and Black Forest cake. The weather was nice during the hike and almost all participants managed to return from the long walk before it got really dark. Overall, the atmosphere was very relaxed and stimulated discussions and free scientific gatherings of the participants – during the last day of the conference a considerable audience gathered into the seminar room to listen to improvised extra lectures given around midnight!

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1641185, “US Junior Oberwolfach Fellows”.
## Table of Contents

Christoph Aistleitner  
*Sums of greatest common divisors and metric number theory* .......... 3039

Régis de la Bretèche (joint with Gérald Tenenbaum)  
*A remark on Sarnak’s conjecture* ........................ 3040

Jean-François Burnol  
*Non linear chains in the study of functional equations* ................. 3042

Adam J. Harper  
*A proof of Helson’s conjecture* .................................. 3044

Titus W. Hilberdink  
*Problems concerning Beurling’s generalized primes* ...................... 3047

Christopher Hughes  
*Random Matrix Theory and the Maximum of the Riemann Zeta Function* 3048

Sofia Lindqvist (joint with Sam Chow, Sean Prendiville)  
*Rado’s criterion for k’th powers* .................................. 3053

Hugh L. Montgomery  
*Mean values of Dirichlet polynomials* ................................ 3054

Hugh L. Montgomery  
*Large values of Dirichlet polynomials* ............................ 3056

Antonio Pérez (joint with Andreas Defant)  
*Optimal comparison of the p-norms of Dirichlet polynomials* .......... 3057

Hervé Queffélec  
*Approximation numbers of composition operators in dimension two or more* .......................................................... 3060

Meredith Sargent  
*Carlson’s Theorem for Different Measures* .......................... 3061

Ingo Schoolmann (joint with Andreas Defant)  
*Hardy type spaces of general Dirichlet series* ........................ 3061

Joni Teräväinen (joint with Terence Tao)  
*On correlations of multiplicative functions* ........................... 3063

Håkan Hedenmalm, Michel Balazard, Titus Hilberdink, Frédéric Bayart, Jean-François Burnol  
*Open problems session* ............................................. 3065