Abstract. Reflectionless operators in one dimension are particularly amenable to inverse scattering and are intimately related to integrable systems like KdV and Toda. Recent work has indicated a strong (but not equivalent) relationship between reflectionless operators and almost periodic potentials with absolutely continuous spectrum. This makes the realm of reflectionless operators a natural place to begin addressing Deift’s conjecture on integrable flows with almost periodic initial conditions and Simon’s conjecture on gems of spectral theory establishing correspondences between certain coefficient and spectral properties.

Mathematics Subject Classification (2010): 35Q53, 35B15, 47B36, 47A55.

Introduction by the Organisers

This mini-workshop was organized by David Damanik (Rice), Fritz Gesztesy (Baylor), and Peter Yuditskii (Linz). The program consisted of 15 lectures on a broad variety of problems related to the Deift and Simon conjectures, including perturbation theory, integrable systems, random matrix theory, character-automorphic Hardy spaces, and orthogonal polynomials. This workshop intended to provide a cutting-edge survey of new results for reflectionless operators, especially those results directed towards addressing Deift’s conjecture regarding the almost periodicity of solutions to the KdV equation with almost-periodic initial data and Simon’s conjecture regarding gems of spectral theory establishing a one-to-one correspondence between suitable classes of coefficient data and spectral data. This
forum provided a great background for discussions of some of the extant open problems in the field.

Sixteen mathematicians took part in this mini-workshop, most of whom traveled from abroad to attend. The organizers and participants would like to extend their sincere gratitude towards the MFO for their hospitality and for providing a beautiful location to discuss and do mathematics.

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Mini-Workshop: Reflectionless Operators: The Deift and Simon Conjectures

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