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Automorphic Forms and Arithmetic

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Abstract. The workshop brought together leading experts and young researchers at the interface of automorphic forms and analytic number theory to disseminate, discuss and develop important recent methods and results. A particular focus was on higher rank groups, as well as their arithmetic applications. This includes, for instance, the study of various aspects of L-functions, the fine distribution properties of their Fourier coefficients and Hecke eigenvalues, the mass distribution of automorphic forms on general symmetric spaces, and applications of results of algebraic geometry to automorphic forms.

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Introduction by the Organisers

Automorphic forms are a very interdisciplinary topic in modern mathematics at the interface of number theory, analysis, representation theory and algebraic geometry. Among these different view points, the workshop focused in particular on the analytic theory of automorphic forms and their associated L-functions, and their interactions with number theory. Fifty-two leading experts and young researchers came together to exchange ideas, present newly developed methods and start or continue their collaboration on projects related to the subject of the workshop. The programme included 25 talks, all of which presented interesting new results. We highlight a few major topics:

The most important global analytic objects attached to an automorphic form or representation f are its L-functions, and they play naturally a major role in

the investigation of automorphic forms as well as in their own interest as a generalization of the Riemann zeta-function. R. Holowinsky presented a new look on Munshi's automorphic circle method, and Munshi presented a new application of it. D. Koukoulopoulos discussed the use of pretentious methods to prove automorphic prime number theorems. M. Radziwiłł explained his work (with collaborators) concerning the challenging arithmetic and analytic problems of evaluation of higher-moments of certain families of L-functions, while O. Balkanova discussed joint work with D. Frolenkov where new analytic techniques are used to evaluate moments of cusp forms in the fixed weight aspect. K. Soundararajan talked about an exciting new method (developed with M. Radziwiłł) that produces effective lower bounds for L-values whenever it is known from an analytic method that they are non-zero. M. Milinovich discussed new results concerning simple zeros of L-functions.

In arithmetic situations, special values of L-functions often encode periods of automorphic forms. One of the most general versions of this principle is the Gross-Prasad conjecture. An important (proven) special case of this concerns triple product L-functions, whose central value encodes information on the mass distribution of an automorphic form. As an analogue of the famous Quantum Unique Ergodicity Conjecture of Rudnick and Sarnak, one can consider the mass distribution of a holomorphic cusp of large weight or level (or both) and deduce in arithmetic situations a weak-star equidistribution result from a subconvexity bound of the corresponding triple product L-functions. P. Nelson presented an ingenious new method that is capable of reducing the problem to a subconvexity bound of degree 3 symmetric square L-functions, which in turn is accessible in a strong quantitative way (with power saving) by Munshi's circle method.

Another way of measuring the mass distribution of an automorphic form is given by the sup-norm, and several talks (by P. Maga, S. Marshall, D. Milićević and A. Saha) focussed on various aspects of this problem. This features a very beautiful combination of arithmetic and analysis and fit therefore nicely into the theme of this conference. The first step in most approaches to the sup-norm problem is an application of the trace formula or at least the spectral expansion of an automorphic kernel, often called a pre-trace formula. Trace formulae belong to the most powerful tools in the theory of automorphic forms. Being a vast generalization of the Poisson summation formula to a non-commutative setting, they translate spectral information of symmetric spaces into algebro-geometric information of the underlying group. Meanwhile the Arthur-Selberg trace formula has been developed to a point where equidistribution questions can be attacked by analytic means. This is an exciting new area with considerable potential, and was the subject of talks by J. Buttcane, J. Matz and M. Young.

A number of talks were devoted to interactions between automorphic methods and problems and various aspects of number theory. Z. Rudnick and W. Sawin presented problems concerning function fields, insisting on interesting phenomena. There were also talks on ergodic techniques (D. Kelmer and M. Lee), transfer operators (A. Pohl), Arakelov theory and invariants (A.-M. von Pippich).

A particularly remarkable application of modular forms to a problem of Fourier approximation on the real line was presented by M. Viazovska.

As fruitful as the talks presented at this workshop were informal discussion after lunch and after dinner that initiated several new projects. This included (on the very last day of the workshop), and following the talk of M. Risager on modular symbols and their distribution, the proof of a conjecture of Mazur and Rubin related to this topic. Wednesday evening was devoted to a lively and interesting problem session.

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