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Computational Inverse Problems for Partial Differential Equations

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Abstract. The problem of determining unknown quantities in a PDE from measurements of (part of) the solution to this PDE arises in a wide range of applications in science, technology, medicine, and finance. The unknown quantity may e.g. be a coefficient, an initial or a boundary condition, a source term, or the shape of a boundary. The identification of such quantities is often computationally challenging and requires profound knowledge of the analytical properties of the underlying PDE as well as numerical techniques. The focus of this workshop was on applications in phase retrieval, imaging with waves in random media, and seismology of the Earth and the Sun, a further emphasis was put on stochastic aspects in the context of uncertainty quantification and parameter identification in stochastic differential equations. Many open problems and mathematical challenges in application fields were addressed, and intensive discussions provided an insight into the high potential of joining deep knowledge in numerical analysis, partial differential equations, and regularization, but also in mathematical statistics, homogenization, optimization, differential geometry, numerical linear algebra, and variational analysis to tackle these challenges.

Mathematics Subject Classification (2010): 35R30, 65J20, 65J22, 35R60, 86A15.

Introduction by the Organisers

The workshop was attended by 49 participants from eight countries (15 from outside Europe), eight of them women, as well as nine of them PhD students and PostDocs. The scientific program consisted of 23 long and 4 short talks with extensive discussions, includig one evening talk after the Wednesday excursion to Sankt Roman.

Additionally, a diverse group of participants joined for regular evening discussions on deep learning in model-based inverse problems. Topics included the potential of these methods to (a) learn better regularization; (b) learn model correction in situations where the forward model is inexact, e.g., due to model reduction.

The talks, which covered a broad range of methods and applications and often lead to lively discussion, were gathering around the following focus areas:

Imaging with waves: The reconstruction of an unknown wave speed inside some domain from boundary observations arises in many applications ranging from geophysical prospection to ultrasound imaging. One issues here is multiple scattering leading to artifacts when using linearized reconstruction algorithms. A novel method that was shown to erase multiple scattering, is based on appropriate construction of exterior initial Cauchy data through scattering control. Another approach presented at the meeting relies on reduced order modeling, which can actually be carried out using the measured data only.

In the context of inverse source problems and inverse scattering in time harmonic wave equations, reconstruction algorithms and stability as well as convergence rate estimates were shown. Some of them resulted from uncertainty principles for certain Fourier type transforms, some of them from a monotonicity relation between sets contained in the support of the scatterer and the Neumann-Dirichlet operator. Others used Tikhonov regularization under appropriate regularity conditions on the solution. In the context of electromagnetic waves, imaging of small scatterers was discussed, with resolution results on the Kirchhoff imaging functionals that extend the scalar acoustic case. An important topic in inverse scattering is the computation of transmission eigenvalues, since they carry information on the refractive index of non-absorbing media. Also forward modeling plays an important role here, e.g. the derivation of appropriate radiation conditions or the characterization of shape derivatives as a crucial prerequisite for computationally solving the inverse problem.

Uncertainty quantification: Incomplete information on the geometry of the domain on which the governing PDE holds, modeling errors due to domain truncation, and noise in the data are examples of sources of uncertainty that propagates into the computational solutions of inverse problems. Bayesian approaches allow to quantify such uncertainties, as was demonstrated in the context of electrical impedance tomography, quantitative photoacoustic tomography, modeling of ice sheet flow, among others. Key challenges in the computational solution of infinite dimensional Bayesian inverse problems governed by PDEs include the choice of appropriate prior distributions, efficient solution of PDE constrained optimization problems for evaluating the MAP estimator, approximation of the posterior covariance, and the treatment of correlated non-zero-mean noise.

Phase retrieval: In many practical inverse scattering and wave imaging applications, only intensity measurements are available, whereas phase information is inaccessible to direct observation. Certain wave imaging tasks allow to exploit illumination and frequency diversity in order to recover complete interferometric data, from which the image can be obtained in a robust manner. In X-ray phase contrast imaging, stability results can be achieved under support constraints on the image. Also uniqueness questions as well as error estimates and explicit reconstruction formulas for phaseless inverse medium scattering problems were discussed.

Stochastic differential equations: A strong link between this focus area and the first one was given by a talk on high-order statistics for the random paraxial wave equation, in which the slowly varying envelope satisfies an SDE, the Itô-Schrödinger equation. This, among other results, leads to a stochastic motivation for correlation based imaging. A rather different field of application is modeling of neural activity of the brain by stochastic PDEs or function valued SDEs, which was presented together with a statistical analysis, including, e.g., the derivation of (approximate) 1-d SDEs for certain quantities of interest.

A couple of talks dealt with the problem of estimating parameters — drift and diffusion — in stochastic differential equations. One of them gave an overview on convergence rates results for drift and/or diffusion estimators in the continuous and high frequency observation regime as compared to the more challenging and so far less explored low frequency regime of coarsely spaced observation times. These observation settings were also considered in a different presentation from a Bayesian perspective. An alternative approach to drift and diffusion estimation that is well suited for the low frequency – actually even for the single time – observation case relies to a reformulation of the problem via the Fokker Planck (or Kolmogorov) equation, a deterministic PDE for the transition density.

Seismic imaging: Extracting information on the interior of the earth from seismic waves is a classical source of inverse problems. Related techniques, in particular full waveform inversions, can be used to reconstruct interior quantities in the Sun such as flows and sound speed using correlation data of the line-of-sight velocities on the Solar surface.

Kirchhoff migration is a standard seismic imaging technique based on a zeroth order imaging functional. Alternative, appropriately constructed first order imaging functionals were shown to allow for better recovery of singularities. Full waveform inversion, requires efficient optimization tools that are capable of avoiding local minima. This can, e.g., be achieved by means of multilevel algorithms, whose stability constants and hence radii of attraction can be controlled by scale and frequency.

Beyond these focus areas we also had a number of talks on mixed topics in computation inverse problems for PDEs such as the *reconstruction of singularities of the conductivity* in electrical impedance tomography, based on complex geometric optics solutions and a radial Fourier transform, that enables to establish and exploit relations to X-ray tomography. Further the inverse problem of *recovering microscale properties of materials from effective or homogenized parameters*, where the spectral measure in the integral representation of the homogenized parameters plays a crucial role was considered. In addition *hybrid techniques* such as photocoustic or acousto-optic imaging that enable enhanced biomedical imaging and lead to coupled systems for different physical quantities like light and sound were discussed.

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