Abstract. The workshop on Mathematics of Quantitative Finance, organised at the Mathematisches Forschungsinstitut Oberwolfach from 26 February to 4 March 2017, focused on cutting edge areas of mathematical finance, with an emphasis on the applicability of the new techniques and models presented by the participants.

Mathematics Subject Classification (2010): 91G10, 91G20, 91G60, 91G80, 60F10, 60G22, 34E05, 60H07, 60H15, 60H10.

Introduction by the Organisers

The field of Quantitative Finance is not owned by mathematics: statistics, computer science, economics and even physics (econophysics) all have contributed and continue to contribute to this field. That said, there is a rich community within mathematics that is devoted to further applications of mathematics to finance. Early on, many efforts went into a proper understanding of the absence of arbitrage in models. The ultimate result in this direction-the fundamental theorem of asset pricing (FTAP) in its most general form due to Delbaen-Schachermayer [7, 8]-is a deep result rooted both in functional analysis and in stochastic analysis. It has been said ironically that depending where one stands this is either the most or least important theorem in the field, and indeed, there are many problems in practice where general results on the absence of arbitrage yield little insight into a concrete problem. Many of the investigations in recent years have been inspired-in more or less direct ways-from such concrete problems. Some, such as optimal order execution (optimal liquidation of large positions under market impact), are closely related to the financial crisis. Others stem from the desire to extract model-free
information from market data only (such as variance swap theory). Again others
aim to understand the impact of model-uncertainty in applications. We could
not possibly attempt to tackle all directions of ongoing research in this meeting,
but we believe it is part of the beauty of this subject that some of the most
important recent developments in the field are inspired quite directly by problems
from industry. The workshop focused on the following such topics and problems:

- For some time, fractional Brownian motion was considered an object of
limited interest in Finance. In essence, this is due to the failure of fBm (with
$H \neq 1/2$) to be a semi-martingale, thereby allowing arbitrage and hence making
it a poor model for a traded asset. However, recent work by Gatheral, Jaisson and
Rosenbaum [14] exhibits strong evidence that volatility is ‘rough’ (an estimate
for SPX volatility actually gives $H \approx 0.14$, which translates to ‘quite rough’,
also note that there are no arbitrage problems for volatility is not a traded asset).
Such volatility regimes also turn out to be most relevant for option pricing, as they can
resolve the long-standing problem of creating (extreme) volatility smile skews, as
seen in markets, impossible to obtain with classical (finite factor Markovian) sto-
chastic volatility models without jumps (in fact, this explosion of the short-time
smile was proved earlier in [1]). We like to see fractional Brownian motion as an
infinite-dimensional Markovian object. Having said that, it is clear that formu-
las and theories developed with great finesse for finite factor Markovian models
should be carried over to this infinite dimensional setting: geometry, analysis and
numerics of infinite dimensional models with components involving, e.g., fractional
Brownian motions or, more generally, rough paths.

- In interest rate markets, in early 2000, a stochastic volatility model (SABR,
for ‘stochastic $\alpha \beta \rho$’) was proposed and quickly became industry standard for its
seemingly miraculous ‘SABR formula’, an explicit expression for implied volatility
bypassing the need for time-consuming Monte Carlo routines. Behind the miracle
are geometric properties of the model and large deviations theory of stochastic
analysis (in the spirit of Varadhan, Molchanov and many others). The decisive link
to implied volatility asymptotics is due to P. Hagan, A. Lesniewski et al. [17, 18].
Many others have explored this and related topics further, in 2014 Comm. Pure
Appl. Math. alone published three on that matter [9, 10]. Still many (functional)
analytic properties of the SABR model or its relatives like (weak) second order
numerical schemes, or FEM formulations, are unclear. Some discussions about
these gaps were led in the meeting, by several leading specialists on that topic [2, 3, 11, 16].

- Non-linear PDE theory made its decisive appearance in Finance. The brilli-
collects a number of ideas useful in practice (typically in form of numerical algo-
rithms), and open many mathematical questions. (A nice example is given by
the McKean-Vlasov based calibration algorithm, the practically important propa-
gation of chaos is far from clear and numerical simulations suggest ‘bad’ regimes
in which propagation of chaos actually fails). Here many aspects are wide open: convergence proofs and rates of convergence including complexity estimates.

- The role of information in stochastic models of Finance has been treated traditionally by filtrations of $\sigma$-algebras. From a practical point of view it is clear that there is a gap between the instantaneous information arriving through price changes or news and the information entering actual trading or investment strategies, i.e. one should split the market’s filtration and the trader’s filtration. To consider this delay or, more generally, information gap, as a fundamental property of markets, i.e. to develop a theory where trading decision are made with respect to smaller filtrations whereas models are written on a larger filtration, in other words a theory of Bayesian Finance, has only been considered in special cases. Actual models, where such structures are reflected, could include recently introduced uncertain volatility models [6]. Remarkably such models behave extremely well from the point of view of calibration, but have never been considered systematically from a fundamental point of view. In particular the corresponding FTAP for Bayesian Finance and its connection to robust Finance is unclear.

- In recent years, particular attention has been given to model-free Finance, where, instead of relying on a specific (class of) model(s), data provides the essential structure of the pricing framework, in the form of lower and upper bounds for option prices. Optimal transport tools therefore received a warm welcome in quantitative finance, allowing us to free ourselves from the sometimes too narrow applicability of models.

These fundamental topics were investigated in detail in the 42 talks (of varied lengths) given by some of the participants during the workshop. As the above research topics illustrate, 20 years after settling the fundamental theorems, the field is sparkling with new ideas from all directions of applied mathematics and beyond.

Peter Friz, Antoine Jacquier, Josef Teichmann

References


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