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Mini-Workshop: Stochastic Differential Equations: Regularity and Numerical Analysis in Finite and Infinite Dimensions

Organised by
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Abstract. This Mini-Workshop is devoted to regularity and numerical analysis of stochastic ordinary and partial differential equations (SDEs for both). The standard assumption in the literature on SDEs is global Lipschitz continuity of the coefficient functions. However, many SDEs arising from applications fail to have globally Lipschitz continuous coefficients. Recent years have seen a prosper growth of the literature on regularity and numerical approximations for SDEs with non-globally Lipschitz coefficients. Some surprising results have been obtained – e.g., the Euler–Maruyama method diverges for a large class of SDEs with super-linearly growing coefficients, and the limiting equation of a spatial discretization of the stochastic Burgers equation depends on whether the discretization is symmetric or not. Several positive results have been obtained. However the regularity of numerous important SDEs and the closely related question of convergence and convergence rates of numerical approximations remain open. The aim of this workshop is to bring together the main contributers in this direction and to foster significant progress.

Mathematics Subject Classification (2010): 65C30, 65C35, 60H15, 65N75.

Introduction by the Organisers

The workshop *Mini-Workshop: Stochastic Differential Equations: Regularity and Numerical Analysis in Finite and Infinite Dimensions* was organised by Martin Hutzenthaler (Universität Duisburg-Essen), Annika Lang (Chalmers University, Göteburg), Lukasz Spruch (University of Edinburgh), and Larisa Yaroslavtseva

(University of Passau). It was attended by 16 participants from France, Germany, the Netherlands, Sweden, Switzerland, and the United Kingdom. Most participants were young researchers working on numerical analysis for 'non-standard' SDEs (i.e., SDEs with coefficients that do not satisfy global Lipschitz or monotonicity conditions).

Denis Talay, Stig Larsson, and Arnulf Jentzen agreed to give overview talks (2x45 minutes), in which they also presented some open problems. Arnulf Jentzen gave an overview of positive and negative results regarding both strong and weak convergence (rates) for approximations of non-linear SDEs. He also posed a great number of open problems of varying (presumed) difficulty. Examples include characterizing the strong/weak convergence rates of the Heston model in terms of the model parameters, and determining optimal weak convergence rates for non-linear stochastic partial differential equations (e.g., the non-linear heat equation). One open problem was also mentioned in the talk of Stig Larsson: it concerns obtaining (optimal) strong convergence rates for temporal discretisations of the Cahn–Hilliard–Cook equation. Stig Larsson discussed the Cahn–Hilliard–Cook equation (physical interpretation, well-posedness) and presented some recent results regarding convergence of numerical schemes for this equation. Denis Talay gave a brief introduction to McKean–Vlasov particle interaction systems with smooth kernels, explaining how — when letting the number of particles involved go to infinity — the solution converges weakly to the solution of the McKean–Vlasov SDE. He then continued to discuss the case of singular kernels and to explain how these arise naturally in certain neurological models, and concluded his talk with a discussion of the occurance of blow-ups in such equations.

All other participants also contributed a talk on recent research questions, and some additionally presented open problems. The small scale of the workshop allowed for a very informal atmosphere, leading to numerous discussions during and after the talks and excellent group dynamics.

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