

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 9/2017

DOI: 10.4171/OWR/2017/9

**Mini-Workshop: Stochastic Differential Equations:  
Regularity and Numerical Analysis in Finite and Infinite  
Dimensions**

Organised by  
Martin Hutzenthaler, Essen  
Annika Lang, Göteborg  
Lukasz Szpruch, Edinburgh  
Larisa Yaroslavtseva, Passau

5 February – 11 February 2017

**ABSTRACT.** This Mini-Workshop is devoted to regularity and numerical analysis of stochastic ordinary and partial differential equations (SDEs for both). The standard assumption in the literature on SDEs is global Lipschitz continuity of the coefficient functions. However, many SDEs arising from applications fail to have globally Lipschitz continuous coefficients. Recent years have seen a prosper growth of the literature on regularity and numerical approximations for SDEs with non-globally Lipschitz coefficients. Some surprising results have been obtained – e.g., the Euler–Maruyama method diverges for a large class of SDEs with super-linearly growing coefficients, and the limiting equation of a spatial discretization of the stochastic Burgers equation depends on whether the discretization is symmetric or not. Several positive results have been obtained. However the regularity of numerous important SDEs and the closely related question of convergence and convergence rates of numerical approximations remain open. The aim of this workshop is to bring together the main contributors in this direction and to foster significant progress.

*Mathematics Subject Classification (2010):* 65C30, 65C35, 60H15, 65N75.

**Introduction by the Organisers**

The workshop *Mini-Workshop: Stochastic Differential Equations: Regularity and Numerical Analysis in Finite and Infinite Dimensions* was organised by Martin Hutzenthaler (Universität Duisburg-Essen), Annika Lang (Chalmers University, Göteborg), Lukasz Spruch (University of Edinburgh), and Larisa Yaroslavtseva

(University of Passau). It was attended by 16 participants from France, Germany, the Netherlands, Sweden, Switzerland, and the United Kingdom. Most participants were young researchers working on numerical analysis for ‘non-standard’ SDEs (i.e., SDEs with coefficients that do not satisfy global Lipschitz or monotonicity conditions).

Denis Talay, Stig Larsson, and Arnulf Jentzen agreed to give overview talks (2x45 minutes), in which they also presented some open problems. Arnulf Jentzen gave an overview of positive and negative results regarding both strong and weak convergence (rates) for approximations of non-linear SDEs. He also posed a great number of open problems of varying (presumed) difficulty. Examples include characterizing the strong/weak convergence rates of the Heston model in terms of the model parameters, and determining optimal weak convergence rates for non-linear stochastic partial differential equations (e.g., the non-linear heat equation). One open problem was also mentioned in the talk of Stig Larsson: it concerns obtaining (optimal) strong convergence rates for temporal discretisations of the Cahn–Hilliard–Cook equation. Stig Larsson discussed the Cahn–Hilliard–Cook equation (physical interpretation, well-posedness) and presented some recent results regarding convergence of numerical schemes for this equation. Denis Talay gave a brief introduction to McKean–Vlasov particle interaction systems with smooth kernels, explaining how — when letting the number of particles involved go to infinity — the solution converges weakly to the solution of the McKean–Vlasov SDE. He then continued to discuss the case of singular kernels and to explain how these arise naturally in certain neurological models, and concluded his talk with a discussion of the occurrence of blow-ups in such equations.

All other participants also contributed a talk on recent research questions, and some additionally presented open problems. The small scale of the workshop allowed for a very informal atmosphere, leading to numerous discussions during and after the talks and excellent group dynamics.

*Acknowledgement:* The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.

## Mini-Workshop: Stochastic Differential Equations: Regularity and Numerical Analysis in Finite and Infinite Dimensions

### Table of Contents

|   |     |
|---|-----|
| Andrea Barth (joint with Andreas Stein)   |     |
| <i>Approximation and simulation of infinite-dimensional Lévy processes</i> . . .  | 457 |
| Lisa Beck (joint with Franco Flandoli, Massimiliano Gubinelli, Mario<br>Maurelli)   |     |
| <i>Regularization by noise for the stochastic transport equation</i> . . . . .  | 459 |
| Dirk Blömker  |     |
| <i>Computable a-posteriori bounds for SPDEs</i> . . . . .   | 461 |
| David Cohen (joint with Rikard Anton)   |     |
| <i>Exponential integrators for stochastic Schrödinger equations driven by Itô<br/>    noise</i> . . . . .   | 463 |
| Sonja Cox (joint with Martin Hutzenthaler, Arnulf Jentzen, Jan van<br>Neerven, Timo Welti)  |     |
| <i>Convergence in Hölder norms with applications to Galerkin<br/>    approximations and Monte Carlo methods</i> . . . . .                                       | 464 |
| Mario Hefter (joint with Arnulf Jentzen)  |     |
| <i>On arbitrarily slow convergence rates for strong numerical approximations<br/>    of Cox–Ingersoll–Ross processes and squared Bessel processes</i> . . . . . | 466 |
| Martin Hutzenthaler (joint with Sonja Cox, Martin Hairer, Arnulf Jentzen,<br>Xiaojie Wang)  |     |
| <i>A perturbation theory and exponential moments for SDEs</i> . . . . .   | 469 |
| Arnulf Jentzen (joint with Máté Gerencsér and Diyora Salimova)  |     |
| <i>A review on stochastic differential equations with arbitrarily slow<br/>    convergence rates for strong approximation in two space dimensions</i> . . . .   | 471 |
| Raphael Kruse   |     |
| <i>On a stochastic version of the Prothero–Robinson problem</i> . . . . .   | 473 |
| Annika Lang (joint with Andreas Petersson, Andreas Thalhammer)  |     |
| <i>Mean-square stability analysis of SPDE approximations</i> . . . . .  | 476 |
| Stig Larsson (joint with Daisuke Furihata, Mihály Kovács, Fredrik<br>Lindgren, and Ali Mesforush)   |     |
| <i>On the convergence of numerical approximations of the stochastic<br/>    Cahn–Hilliard equation</i> . . . . .  | 480 |

---

|   |     |
|---|-----|
| Annie Millet (joint with Hakima Bessaih)  |     |
| <i>On stochastic Brinkman–Forchheimer anisotropic 3D Navier–Stokes equations</i> .....                      | 483 |
| Andreas Neuenkirch (joint with Taras Shalaiko)  |     |
| <i>Simulating rough volatility models</i> .....   | 486 |
| Lukasz Szpruch (joint with Shuren Tan)  |     |
| <i>Multilevel Monte Carlo for McKean–Vlasov SDEs.</i> .....   | 489 |
| Denis Talay   |     |
| <i>On some singular McKean–Vlasov particle systems</i> .....  | 490 |
| Larisa Yaroslavtseva (joint with Arnulf Jentzen, Thomas Müller-Gronbach)                                    |     |
| <i>On sub-polynomial lower error bounds for strong approximation of SDEs with smooth coefficients</i> ..... | 493 |