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Combinatorics

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ABSTRACT. Combinatorics is a fundamental mathematical discipline that focuses on the study of discrete objects and their properties. The present workshop featured research in such diverse areas as Extremal, Probabilistic and Algebraic Combinatorics, Graph Theory, Discrete Geometry, Combinatorial Optimization, Theory of Computation and Statistical Mechanics. It provided current accounts of exciting developments and challenges in these fields and a stimulating venue for a variety of fruitful interactions. This is a report on the meeting, containing abstracts of the presentations and a summary of the problem session.

Mathematics Subject Classification (2010): 05-XX.

Introduction by the Organisers

The workshop Combinatorics, organized by Jeff Kahn (Piscataway), Angelika Steger (Zürich) and Benny Sudakov (Zürich), was held the first week of January, 2017. Despite the early point in the year the meeting was well attended, with roughly 50 participants from the US, Canada, Brazil, UK, Israel, and various European countries. The program consisted of 11 plenary lectures and 18 shorter contributions, including the presentations by Oberwolfach Leibniz graduate students. There was also a lively problem session led by Nati Linial. The plenary lectures were chosen to provide both overviews of the state of the art in various areas and in-depth treatments of major new results. The short talks ranged over a broad range of topics, including, *for example* (far from an exhaustive list), graph theory, coding theory, probabilistic combinatorics, discrete geometry, extremal combinatorics and Ramsey theory, additive combinatorics, and theoretical computer science. As in the

past, particular stress was placed on providing a platform for younger researchers to present themselves and their results.

This report contains extended abstracts of the talks and statements of problems that were posed at the problem session. This was a particularly successful edition of the meeting Combinatorics, in large part because of the exceptional strength and range of both the participants and the results presented. While it is hard to do justice to a meeting at such a level in this short summary, we here highlight just two of the more spectacular developments.

The first of these is Ron Peled’s resolution (with Yinon Spinka) of a more than 30-year-old conjecture of R. Kotecký on the antiferromagnetic Potts model. (Their results apply in the far more general context of *graph homomorphisms*, but here we only discuss the special case that was the subject of R. Kotecký’s conjecture.)

For a fixed $q \geq 3$, large fixed d , and very large n , we consider σ chosen uniformly at random from the set of proper q -colorings of the “box”

$$\Lambda_n^d = \{x \in \mathbb{Z}^d : \|x\|_\infty \leq n\}.$$

In the language of statistical mechanics this is the *antiferromagnetic Potts model at temperature zero*. Kotecký conjectured (circa 1985) an affirmative answer to the basic (“phase transition” or, more properly, “phase coexistence”) question for this model; namely, *does the model admit multiple maximal entropy Gibbs measures?* Such an answer is supplied by the following combinatorial statement, proved by Peled and Spinka, in which ∂ denotes boundary (thus $\partial\Lambda_n^d = \{x \in \mathbb{Z}^d : \|x\|_\infty = n\}$).

Theorem. *If $A \cup B$ is an equipartition of $\{1, \dots, q\}$ (that is, $\|A\| - \|B\| \leq 1$) and we condition on σ assigning colors from A (resp. B) to all even (resp. odd) vertices of $\partial\Lambda_n^d$, then*

$$\Pr(\sigma(\underline{0}) \in A) > 1 - \epsilon_d,$$

where $\epsilon_d \rightarrow 0$ as $d \rightarrow \infty$.

That is: in high enough (fixed) dimension, the influence of the boundary conditions on behavior at the origin persists as the boundary recedes. For $q = 3$ this had been known for about 15 years and stronger results have more recently been established by Peled and others; but the jump to $q \geq 4$ had proved wholly intractable until this breakthrough.

Our second highlight is the work discussed by Daniela Kühn (obtained jointly with Stefan Glock, Allan Lo and Deryk Osthus), giving a “second generation” proof of a vast generalization of the “Existence Conjecture” for block designs. (This famous nineteenth century problem was settled a few years ago by Peter Keevash and first announced at the January 2014 edition of “Combinatorics.”) To give some flavor, we just state (skipping some specifics) a special case of the original conjecture:

For fixed $2 \leq t \leq k$ and any sufficiently large n obeying some obviously necessary congruential restrictions, it is possible to find a collection of k -subsets of $\{1, \dots, n\}$ containing each t -element set exactly once.

For $t = 2$ this was proved in celebrated work of R.M. Wilson in the early 1970's, but there was really nothing between Wilson and Keevash that could be called serious progress on this basic problem; indeed—perhaps most strikingly—for $t \geq 6$ *not a single example of such a collection was known to exist.*

Keevash's work, based on a new notion of “randomized algebraic construction,” was a complete departure from the methods that have dominated design theory for the last forty years. (Recent times have seen a number of such examples; that is, of celebrated problems that had usually been attacked by other methods succumbing to ideas more prevalent in the community at the heart of the present meeting.)

The formidable work of Kühn *et al.* goes well beyond Keevash, whose approach already encompasses much more than the Existence Conjecture (as indeed it must to succeed). Here the algebraic aspects of Keevash are replaced by a highly sophisticated (“iterative”) version of the “absorbing method” of Rödl, Ruciński and Szemerédi, and the work may be considered a sort of culmination (at least to date) in the development of this powerful approach. A description of even a representative subset of the new results seems infeasible for the present summary. The new, more purely combinatorial approach provides considerable flexibility that looks sure to lead to many further developments.

As always, and on behalf of all participants, the organizers would like to thank the staff and the director of the Mathematisches Forschungsinstitut Oberwolfach for providing such a stimulating and inspiring atmosphere.

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Workshop: Combinatorics

Table of Contents

Igor Balla (joint with F. Draxler, P. Keevash, and B. Sudakov) <i>Equiangular lines and spherical codes</i>	11
József Balogh (joint with J. Solymosi) <i>On the number of points in general position in the plane</i>	14
Alexander Barvinok <i>Permanents and complex zeros</i>	16
Boris Bukh (joint with G. Nivasch) <i>One-sided epsilon-approximants</i>	17
Maria Chudnovsky (joint with A. Scott, P. Seymour, and S. Spirkl) <i>Induced subgraphs and chromatic number</i>	19
David Conlon (joint with A. Ferber, R. Nenadov, and N. Škorić) <i>Universality-type results in random graphs</i>	21
Asaf Ferber (joint with A. Bandeira and M. Kwan) <i>On a resilience Littlewood-Offord problem</i>	22
Jacob Fox (joint with L. M. Lovasz, H. T. Phan, and Y. Zhao) <i>Arithmetic progressions, regularity, and removal</i>	25
Zoltán Füredi (joint with I. Kantor) <i>Kneser ranks of random graphs and minimum difference representations</i>	29
David Gamarnik (joint with M. Sudan) <i>Performance of Sequential Local Algorithms for the Random NAE-K-SAT Problem</i>	32
Peter Keevash (joint with E. Long) <i>Forbidden vector-valued intersections</i>	36
Swastik Kopparty (joint with J. Gilmer) <i>The distribution of the number of triangles in a random graph</i>	40
Daniel Král' (joint with R. Glebov, A. Grzesik, P. Hu, T. Hubai, J. Volec) <i>Local 3-profiles of graphs</i>	41
Daniela Kühn (joint with S. Glock, A. Lo, and D. Osthus) <i>The existence of designs – beyond quasirandomness</i>	42
Nati Linial <i>Some stories of high-dimensional permutations</i>	45

Tomasz Łuczak (joint with Y. Peled)	
<i>Integral homology of random simplicial complexes</i>	46
Roy Meshulam (joint with A. Lubotzky)	
<i>High Dimensional Expansion</i>	48
Rob Morris (joint with P. Balister and B. Bollobás)	
<i>The sharp threshold for making squares</i>	50
Frank Mousset (joint with A. Noever, K. Panagiotou, and W. Samotij)	
<i>Higher-order terms in Janson's inequality</i>	53
János Pach (joint with J. Fox and A. Suk)	
<i>Estimating the rectilinear crossing number of dense graphs</i>	55
Ron Peled (joint with Y. Spinka)	
<i>Long-range order in random colorings and graph homomorphisms</i>	56
Will Perkins (joint with E. Davies, M. Jenssen, and B. Roberts)	
<i>An occupancy approach to bounding graph polynomials</i>	59
Christian Reiher (joint with V. Rödl, A. Ruciński, M. Schacht, E. Szemerédi)	
<i>Hamiltonian cycles in 3-uniform hypergraphs</i>	62
Shubhangi Saraf (joint with A. Basit, Z. Dvir, and C. Wolf)	
<i>Incidence geometry, rank bounds for design matrices, and applications</i> .	65
Asaf Shapira (joint with L. Gishboliner)	
<i>Removal Lemmas with Polynomial Bounds</i>	66
Kathrin Skubch (joint with A. Coja-Oghlan, O. Cooley, and M. Kang)	
<i>Core Forging by Warning Propagation</i>	69
József Solymosi (joint with J. Ellenberg and J. Zahl)	
<i>An application of the polynomial method</i>	70
Jacques Verstraëte (joint with M. Molloy and B. Sudakov)	
<i>Generalized Turán Numbers</i>	71
Yufei Zhao (joint with B. Bhattacharya, S. Ganguly, and X. Shao)	
<i>Upper tails for arithmetic progressions</i>	73
<i>Problem Session</i>	75