Abstract. The efficient numerical solution of highly oscillatory problems is one of the grand challenges of Applied Mathematics with diverse applications across the natural sciences and engineering. This workshop brings together experts in domain based methods and integral equation methods to share novel ideas and to discuss challenges on the way to developing efficient solvers at high frequencies.

Mathematics Subject Classification (2010): 65xx.

Introduction by the Organisers

The fast solution of highly oscillatory problems remains one of the great challenges of applied and computational mathematics. This workshop brought together experts working on fast direct solvers for integral equations, preconditioning and domain decomposition methods to share novel ideas for the development of scalable frequency domain solvers for acoustic and electromagnetic problems.

The workshop was roughly divided into three broad subject areas, namely 1.) fast direct solvers for Helmholtz problems, 2.) fast iterative methods and preconditioning for oscillatory integral equations, and 3.) domain decomposition methods for volume problems.

The first day started off with an overview talk by Per-Gunnar Martinsson, outlining the challenges of developing fast direct solvers for high-frequency problems. We then had talks by Steffen Börm and Markus Melenk on novel directional $H^2$ matrix techniques for highly oscillatory problems.
In the evening Timo Betcke led a discussion on large-scale industrial challenges for high-frequency solvers and the need to develop large-scale coupled FEM/BEM domain decomposition frameworks to address them.

The second day saw talks by Adrianna Gillman and Alex Barnett on fast direct solvers for oscillatory problems, and on the fast solution of periodic problems, respectively. The algorithms presented in these talks produced stunning results and were backed up by beautiful graphical visualizations of solutions of oscillatory problems in two and three space dimensions (see also the respective extended abstracts).

Significant discussions were created by Mike O’Neil’s talk. He presented novel results on butterfly algorithms, and it was decided to devote the whole Wednesday afternoon to a more detailed understanding of butterfly algorithms. Butterfly compression has the potential to significantly improve the efficiency of fast direct solvers for oscillatory problems and a lot of work is currently going into the development of novel algorithms based on butterfly ideas.

A remarkable result of the butterfly discussions during the week was that directional $H^2$ structures applied to individual admissible blocks lead to a butterfly representation. This opens up the potential to apply algorithmic developments for $H^2$ matrices to butterfly decompositions.

Due to weather changes the traditional tour was done together with the other workshop groups already on Tuesday afternoon.

On Wednesday the focus shifted to fast multipole methods and preconditioning. Stéphanie Chaillat started with an overview talk on Fast Multipole Methods and various applications in elastodynamics, followed by a talk by Marion Darbas on novel analytic preconditioners for high-frequency elastic problems.

The talks were concluded on Wednesday by an overview presentation by Timo Betcke on the BEM++ software framework which provides solvers for a wide range of electrostatic, acoustic and electromagnetic problems. In the afternoon the aforementioned discussions on butterfly algorithms, led by Mike O’Neil took place.

Thursday started with the second part of Stéphanie Chaillat’s overview talk on fast solvers for elastodynamics. This was followed by an overview by Sabine Le Borne on the use of fast hierarchical matrix solver techniques for integral equations in scattered data approximation problems.

The final talk of the day was an overview talk by Martin Gander on variants of optimized Schwarz domain decomposition solvers for high-frequency problems. His framework generalizes a range of methods, including sweeping preconditioners, polarized traces, and multitrace formulations. This sparked many discussions in the afternoon, leading over to Friday which concluded with talks by Ivan Graham on shifted Laplacian preconditioners and an introduction to the ideas behind polarized traces by Laurent Demanet, which provided a fitting conclusion to the workshop.

The workshop created a unique atmosphere to bring together fast solver experts from the domain decomposition and the boundary integral equation community.
The main threads that developed throughout the workshop were the efficient use of directional approximations and butterfly ideas, numerical and analytic DtN approximations for preconditioning and as transmission conditions in domain decomposition methods, and unifying domain decomposition frameworks that can incorporate a range of currently investigated methods. The format of the workshop allowed to exchange these ideas and give strong impulses for future research into fast high-frequency solvers.

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