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Theory and Numerics of Inverse Scattering Problems

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Abstract. This workshop addressed specific inverse problems for the time-harmonic Maxwell's equations, resp. special cases of these, such as the Helmholtz equation or quasistatic approximations like in impedance tomography. The inverse problems considered include the reconstruction of obstacles and/or their material properties in a known background, given various kinds of data, such as near or far field measurements in the scattering context and boundary measurements in the quasistatic case.

Mathematics Subject Classification (2010): 35J05, 31B20, 65N20.

Introduction by the Organisers

Inverse scattering problems are of fundamental practical relevance, as the sensing of scattered waves is one of our most profound interactions with our neighborhood. Extracting information about the unknown medium or the force that is generating the scattered wave is often also a question of high economical value.

To address these problems basic mathematical questions need to be answered, including, for example, the following ones:

- What data are required to determine the shape of a scattering object (uniqueness problem)?
- Is there a constructive method to do so?

This workshop brought together 26 participants from seven different countries from all over the world, including both reknowned international experts as well as promising young postdocs and PhD students. The contributed presentations were

mostly dealing with inverse scattering problems and electrical impedance tomography, but there also have been talks on related topics such as elastodynamics and optical coherence tomography. Although the primary focus of the meeting was on inverse problems there have also been contributed talks on the proper (well-posed) formulation of the associated direct or forward problems, and on corresponding existence and uniqueness theorems.

The majority of talks was addressing latest progress for the classical inverse obstacle scattering problem, which comes in various flavors depending on the material properties of the scatterer. These material properties can affect the boundary conditions as well as the index of refraction in the interior of the scatterer. Concerning the latter, a previous Oberwolfach meeting in 2012 had boosted the momentum of the theoretical investigation of so-called *transmission eigenvalues*, i.e., specific frequencies at which a certain obstacle or inhomogeneous medium is invisible for a given primary excitation. While the theory of this highly nonstandard eigenvalue problem was at its infancy at that meeting, there have meanwhile been many new developments in this area. A number of participants, being the main players in this area, gave fascinating insight into the possibilities that arise for the solution of the inverse problems from a computation of these (or related) eigenvalues from the data at hand. For the case of multiple scatterers alternative methods have been proposed to split the measured data into the corresponding components from the individual scatterers first, and then resort to any of the other methods for the full reconstruction.

As far as impedance tomography has been concerned a promising new technique was presented on how to see singularities of the conductivity with a backprojection type method, while another talk demonstrated the dramatic influence of using wrong assumptions about the geometry of the boundary of the object which is being imaged.

Two talks were drawing connections between classical regularization theory and inverse scattering and impedance tomography, respectively. In these presentations rigorous convergence proofs of the numerical reconstructions were discussed when the noise level in the data is going down to zero. Still, being severely ill-posed problems, the corresponding stability estimates are only of logarithmic type.

The meeting also saw an increasing number of contributions addressing (closed or open) wave guides. For the corresponding applications there are still fundamental problems in formulating and numerically solving the direct problems, but due to recent progress it is now possible to also rigorously address the associated inverse problems.

Having been a half-size workshop the schedule left enough space for very interesting and intensive discussions between and after the presentations. At three of the evenings some of the participants came with a glass of wine or beer to join spontaneously organized “informal presentations” of newly emerging fascinating inverse problems in different areas.

In the end all participants including those that could not present a talk for the lack of time agreed to have enjoyed a very special and informative meeting,

and we all look forward to reconvene at this wonderful spot for a similar event. Of course, the splendid support by service and staff was also responsible for this positive reception of the particpants and the great success of this workshop.

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