Mathematisches Forschungsinstitut Oberwolfach

Report No. 43/2016

DOI: 10.4171/OWR/2016/43

Many-Body Quantum Systems and Effective Theories

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11 September – 17 September 2016

Abstract. In the last years, substantial progress has been made in many areas of mathematical physics. The goal of this workshop was to bring together researchers working on analytic and probabilistic aspects of many-body quantum systems and quantum statistical mechanics, to discuss recent developments, exchange ideas and propose new challenges and research directions. Among the questions addressed during the workshop were the derivation of effective equations, the analysis of physically interesting nonlinear partial differential equations emerging from microscopic theories, the study of open quantum systems in and out of equilibrium, and the investigation of the ground state properties and of the dynamics of quantum spin systems.

Mathematics Subject Classification (2010): 82B10, 82C10, 81Q05, 81Q10, 35Q40, 35Q20, 35Q41, 35Q55, 35Q56.

Introduction by the Organisers

During the workshop there were 21 lectures, ranging in topic from the mathematical description of many-particle systems of fermions, bosons and anyons, quantum spin systems, topological phases of matter and the non-equilibrium statistical mechanics of open quantum systems. In addition, there were evening sessions on open problems in the field, which resulted in lively discussions. The first was given on Monday evening by Bruno Nachtergaele, covering quantum spin systems, in particular the existence of spectral gaps, in general for dimensions greater than two and in certain one-dimensional models, e.g. the Haldane spin chain. The second was given by Jan Philip Solovej on Thursday, and concerned open problems in atomic physics, in particular the ionization conjecture and other conjectures concerning the size of atoms with large nuclear charge and the connection to Thomas-Fermi theory.

The workshop covered the following subjects, all of which have received a lot of attention in the mathematical physics community in the last years.

1. Derivation of effective theories

One of the most important questions of statistical mechanics is the justification of effective theories, like Thomas-Fermi, Vlasov and BCS theories, starting from a microscopic description of many-body systems. In this line of research, Mathieu Lewin presented a new general approach to understand the ground state energy of fermionic systems in the mean-field limit, generalizing previous results restricted to very special cases, like systems with Coulomb interaction. Michael Sigal discussed the existence of vortices and vortex lattices in superconductors described by the Bogoliubov-de-Gennes equations. Nicolas Rougerie presented a derivation of effective many-body anyonic systems starting from a model of fermions, where the anyonic degrees of freedom effectively arise in a suitable parameter regime. Alissa Geisinger discussed the question of translational symmetry breaking in the BCS model.

2. Analysis of quantum many-body systems

The focus here is on the analysis of physically interesting properties of many-body systems, described either on the microscopic level, or through approximate theories. For example, Heinz Siedentop presented a derivation of the Scott correction for large atoms for a pseudo-relativistic model of atoms. Ian Jauslin explained a renormalization-group construction of bilayer graphene. Jakob Yngvason discussed his recent work on the incompressibility of Laughlin-type wave functions. Anyons were also the topic of Douglas Lundholm's lecture, in particular Lieb-Thirring inequalities and their dependence on the statistics parameter. Phan Thanh Nam discussed results on the ionization conjecture for various effective theories of atoms, including the Thomas-Fermi-Dirac-von-Weizsäcker theory and the Müller density matrix functional. Stefan Teufel presented an alternative view on particular annihilation, described via boundary conditions on Fock space.

3. Mathematical analysis of nonlinear partial differential equations

Nonlinear partial differential equations arising in the description of many-body quantum systems are the source of several challenging mathematical problems. In the workshop, we had three talks addressing such questions. Enno Lenzmann discussed the existence of solutions of the half-wave-map, an analogue of the wave map with the Laplacian replaced by its square root. Julien Sabin explained the extension of Strichartz inequalities to systems of orthonormal functions. Rupert Frank analyzed the phase transition occurring in a certain model of flocking.

4. Open quantum systems

New phenomena emerge when quantum systems are coupled to large reservoirs, allowing energy and particles to flow. In this setting, Gian Michele Graf discussed the adiabatic crossing of infinitely degenerate energy levels and applications to open quantum systems. Marco Merkli presented applications of the spin-boson model in quantum biology. Wojtech De Roeck explained the effects of periodic driving on quantum systems, and Annalisa Panati discussed energy fluctuations and the full counting statistics.

5. Quantum spin systems

Quantum Spins systems provide toy models in statistical mechanics, which are analytically easier to handle than full many-body theories but, at the same time, capture many of the essential features, like phase transitions, long-range order, correlations and entanglement. They were the subject of two talks in the workshop. Daniel Ueltschi presented new correlations inequalities for the quantum XY model. Yoshiko Ogata explained her work on the classification of frustration free one-dimensional Hamiltonian and corresponding quantum phases.

6. Other

Finally, two talks addressed topics which do not fit precisely in the previous sections but, nevertheless, were related to many-body quantum systems. Felix Finster gave an overview of his work on the causal action principle, in an attempt to try to connect his theory to more traditional approaches. Jonas Lampart discussed the applicability of density functional theory in the time-dependent setting.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, "US Junior Oberwolfach Fellows".

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