

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 38/2016

DOI: 10.4171/OWR/2016/38

Arithmetic Geometry

Organised by
Gerd Faltings, Bonn
Johan de Jong, New York
Peter Scholze, Bonn

7 August – 13 August 2016

ABSTRACT. Arithmetic geometry is at the interface between algebraic geometry and number theory, and studies schemes over the ring of integers of number fields, or their p -adic completions. An emphasis of the workshop was on p -adic techniques, but various other aspects including Hodge theory, Arakelov theory and global questions were discussed.

Mathematics Subject Classification (2010): 11G99.

Introduction by the Organisers

The workshop *Arithmetic Geometry* was well attended by over 50 participants from various backgrounds. It covered a wide range of topics in algebraic geometry and number theory, with some focus on p -adic questions.

Using the theory of perfectoid spaces and related techniques, a number of results have been proved in recent years. At the conference, Caraiani, Gabber, Hansen and Liu reported on such results. In particular, Liu explained general p -adic versions of the Riemann–Hilbert and Simpson correspondences, and Caraiani reported on results on the torsion in the cohomology of Shimura varieties. This involved the geometry of the Hodge–Tate period map, which Hansen extended to a general Shimura variety, using the results reported by Liu. Moreover, Gabber proved degeneration of the Hodge spectral sequence for all proper smooth rigid spaces over nonarchimedean fields of characteristic 0, or even in families, by proving a spreading out result for proper rigid spaces to reduce to a recent result in p -adic Hodge theory.

Another recurring theme was the theory of p -adic families of automorphic forms, and the corresponding Banach representations of p -adic groups, in the framework

of the p -adic local Langlands program. This includes the talks of Andreatta, Colmez and Paškūnas.

In the general framework of p -adic cohomologies, we also had a talk of Jannsen on duality in de Rham–Witt cohomology. Another p -adic topic was Zhang’s talk about the Poisson equation on Berkovich spaces, which is expected to have important applications to the study of heights of rational points.

Large progress in recent years was made on K3 surfaces over finite fields, where many results including the Tate conjecture and the unirationality conjecture have been proved, as well as new results on derived equivalences of K3 surfaces, as reported by Liedtke, and Olsson.

A striking recent result of Abe is the existence of p -adic companions on curves over finite fields, extending the result of L. Lafforgue, attaching ℓ -adic sheaves to automorphic representations, to p -adic cohomology. At the conference, Kedlaya gave a talk on the existence of companions for higher-dimensional varieties, by reduction to the case of curves.

For applications of Arakelov geometry, it is important to control certain contributions coming from the archimedean places. Here, Wilms presented new results on Faltings’s δ -invariant.

Litt has obtained results on the monodromy representations arising from geometry by using anabelian methods and a reduction to characteristic p argument.

Other topics covered were the Grothendieck–Katz p -curvature conjecture by Kisin, the Sansuc formula for general groups by Conrad, Hodge theory and atypical intersections by Klingler, the construction of G_2 -local systems on curves by Katz, and generic vanishing theorems by Bhatt.

During the conference, many active discussions took place. In particular, André finished his proof of the direct summand conjecture, but it was unfortunately too soon to report on it at the meeting.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.

Workshop: Arithmetic Geometry**Table of Contents**

Robert Wilms	
<i>On the delta invariant in Arakelov geometry</i>	2175
Ana Caraiani (joint with Peter Scholze)	
<i>On the generic part of the cohomology of compact unitary Shimura varieties</i>	2178
Christian Liedtke	
<i>Moduli of Supersingular K3 Crystals</i>	2181
Fabrizio Andreatta (joint with Adrian Iovita, Vincent Pilloni)	
<i>The Spectral Halo</i>	2182
Ruochuan Liu (joint with Xinwen Zhu)	
<i>Rigidity and a Riemann-Hilbert correspondence for p-adic local systems</i>	2185
David Hansen	
<i>p-adic period maps and variations of p-adic Hodge structures</i>	2188
Brian Conrad	
<i>Sansuc's Formula and Tate Global Duality (d'après Rosengarten)</i>	2191
Shou-Wu Zhang	
<i>Poisson's equations on Berkovich spaces</i>	2195
Ofer Gabber (joint with Brian Conrad)	
<i>Spreading-out for families of rigid analytic spaces</i>	2197
Daniel Litt	
<i>Arithmetic Restrictions on Geometric Monodromy</i>	2198
Pierre Colmez	
<i>Cohomologie p-adique de la tour de Drinfeld: le cas de dimension 1</i>	2201
Kiran S. Kedlaya	
<i>Étale and crystalline companions</i>	2203
Vytautas Paškūnas (joint with Ana Caraiani, Matthew Emerton, Toby Gee, David Geraghty, Sug Woo Shin)	
<i>Patching and the p-adic Langlands correspondence for $GL_2(\mathbb{Q}_p)$</i>	2204
Uwe Jannsen (joint with Shuji Saito, Yigeng Zhao)	
<i>Filtered de Rham-Witt complexes and wildly ramified higher class field theory over finite fields</i>	2207
Bhargav Bhatt (joint with Christian Schnell, Peter Scholze)	
<i>Generic vanishing for constructible sheaves, revisited</i>	2210

Bruno Klingler	
<i>Hodge theory and atypical intersections</i>	2213
Martin Olsson (joint with Max Lieblich)	
<i>Derived equivalences for K3 surfaces and motives</i>	2216
Mark Kisin (joint with Hélène Esnault)	
<i>D-modules in characteristic p and stability</i>	2217
Nicholas M. Katz	
<i>G_2 and some exceptional Witt Vector identities</i>	2218