

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 28/2016

DOI: 10.4171/OWR/2016/28

## Geometrie

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5 June – 11 June 2016

ABSTRACT. The workshop *Geometry*, organized by John Lott (Berkeley), André Neves (London), Iskander Taimanov (Novosibirsk) and Burkhard Wilking (Münster) was well attended with over 53 participants with broad geographic representation from all continents. Compared to previous meetings there were for example quite a few young Brazilian postdocs at the meeting. The emphasize on min-max problems and related fields was somewhat increased.

*Mathematics Subject Classification (2010)*: 51-XX.

### Introduction by the Organisers

The format of the meeting consisted of 18 one hour talks and four half hour after-dinner talks. The after-dinner talks were given by PhD students and recent PhDs. The schedule left lots of room for discussions in between talks.

Six of the talks were related to geometric flows. Gerhard Huisken investigated the mean curvature flow with surgery in 3-dimensional manifolds. Carlo Sinestrari reported on progress on convergence results of the volume preserving curvature flow for hypersurfaces in Euclidean space. Ramiro Lafuente studied the Ricci flow of homogeneous spaces resulting in a nearly fully understanding of the dynamical properties of the ODE in the solvable case. Renato Bettiol presented some calculations on the Ricci flow on 4-dimensional cohomogeneity one manifolds. Felix Schulze explained some new existence results for Ricci flows coming out of singular spaces which has been a recurrent theme in previous workshops. Namely, he considered as initial space a singular Riemannian manifold with isolated conical

singularities. Finally, Robert Haslhofer explained how the mean curvature flow with surgery can be used to show that the moduli space of 2-convex embedded spheres in Euclidean space is connected.

Two talks generalized well-known results of Yau from the (smooth) Kähler case to a more general setting. Song Sun discussed how one can generalize Yau's solution of the Calabi conjecture to certain singular Calabi Yau varieties. Ben Weinkove discussed the complex Monge-Ampere equations of Hermitian, Gauduchon and balanced metrics.

There were 4 talks involving min-max methods. Yevgeny Liokumovich presented an analogue of Weyl's law for the spectrum of the Laplacian of Riemannian manifolds. Therein the  $p$ -th eigenvalue of the Laplacian is replaced by the  $p$ -width of the manifold, the volume of a minimal hypersurface obtained by a min-max method applied to the  $p$ -th cohomology group of the space of  $(n - 1)$  cycles in the underlying manifold. Rafael Montezuma used min-max methods to construct minimal hypersurfaces in certain noncompact manifolds. Nicolau Sarquis Aiex addressed the question whether for a manifold with an analytic metric of positive Ricci curvature the space of embedded minimal hypersurfaces is non-compact. Daniel Ketover explained how min-max methods can be used to explain the existence of a sequence of minimal surfaces in  $S^3$  converging to the double of the Clifford torus.

An important problem in constructing minimal hypersurfaces using min-max methods is to establish and use index estimates, which three speakers addressed. Alessandro Carlotto gave effective index estimates of minimal hypersurfaces via Euclidean isometric embeddings.IVALDO NUNES reported on stable constant mean curvature surfaces with free boundary. He ruled a potentially exceptional case from earlier work. Davi Maximo investigated the compactness properties of minimal surfaces in Euclidean space with an a priori bound on the index. Roughly they are the same as in the stable case except there are possibly finitely many exceptional points where the convergence is weaker.

Compactness and convergence problems for a sequence of manifolds were at the core of two other talks. Dorothea Jansen investigated collapsing sequences of manifolds with lower Ricci curvature bound. Despite the fact that no form of a fibration theorem is available she showed that the diameter of a typical fiber is well defined up to some uniform factor. Shouhei Honda explained that for a converging sequence of noncollapsed manifolds with bounded Ricci curvature, the spectrum of the Hodge-Laplacian on 1-forms converges as well.

Low eigenvalues of the Laplace operator played an essential role in two talks. Ursula Hamenstädt addressed the several questions in how the first eigenvalue of a hyperbolic three-manifold relates to its volume and its Heegaard genus. Guofang Wei proved optimal gap estimates between the first two eigenvalues of the Laplacian for a convex domain of the sphere.

The remaining three talks were given by Kerin, Hingston and Wickramasekera. Martin Kerin showed that each of the 28 oriented diffeomorphism classes of 7-dimensional spheres admits a metric with nonnegative sectional curvature, which

has previously only been known for those which are  $S^3$ -bundles over  $S^4$ . A crucial step of the proof is to show that each exotic 7-sphere is the total space of a Seifert  $S^3$ -bundle over  $S^4$  endowed with an orbifold metric. Nancy Hingston reported on various loop products with applications to bounds for the number of closed geodesics. Neshan Wickramasekera presented a regularity and compactness theory of CMC hypersurfaces in Riemannian manifolds.

*Acknowledgement:* The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.



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