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# Moduli spaces and Modular forms

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Abstract. The roots of both moduli spaces and modular forms go back to the theory of elliptic curves in the 19th century. Both topics have seen an enormous growth in the second half of the 20th century, but the interaction between the two remained limited. Recently there have been new developments that led to new points of contact between the two topics. One is the theory of K3 surfaces that is rapidly gaining a lot of new interest. Here the link with modular forms on orthogonal groups has led to progress on the Kodaira dimension of the moduli spaces of K3 surfaces. Another new development has been the use of moduli spaces of curves to gather new information about Siegel modular forms. The workshop intended to bring representatives from both the theory of moduli and the theory of modular forms together to further the interaction between the two topics as the time seemed ripe to do this.

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## Introduction by the Organisers

The workshop *Moduli Spaces and Modular Forms*, organized by Jan Bruinier (Darmstadt), Gerard van der Geer (Amsterdam) and Valéry Gritsenko (Lille) was held 25-29 April, 2016 and was attended by 52 participants from all over the world. The attendance ranged from senior leaders in the field to young postdocs and advanced Ph.D. students. The program consisted of 21 talks of one hour or 50 minutes. The lectures and the simple fact that people from different fields were brought together initiated lots of discussions and forged new contacts between participants. The program highlighted the diversity of the interactions between

'Moduli' and 'Modular Forms'. Topics ranged from the sphere packing problem to moduli of supersingular K3 surfaces and Enriques surfaces in characteristic 2.

Three main themes of the workshop were 'Moduli of K3 surfaces and Modular Forms on Orthogonal Groups,' 'Moduli of Curves and Siegel Modular Forms' and 'Modular Forms on Ball Quotients.'

In recent years there has been a strong revival of interest in moduli of K3 surfaces. One development was the determination of the Kodaira dimension for moduli of K3 surfaces of not too small degree, which was the last open problem in A. Weil's program on K3 surfaces. This progress used modular forms on orthogonal groups and Borcherds' automorphic products in an essential way. Another development was the proof of the conjectures of Artin and Tate for K3 surfaces over finite fields for characteristic not 2 last year. Also this proof uses modular forms. Besides this there are interesting developments concerning the compactification of moduli of K3 surfaces. Also the moduli of polarized hyperkähler varieties and Enriques surfaces are attracting new interest in algebraic and differential geometry. All these topics are related to modular forms on orthogonal groups. Apart from this there are interesting links between moduli of K3 surfaces and moduli of curves in a number of papers by Kondo, Allcock and others and modular forms on ball quotients. The modular forms on ball quotients belong to the theory of authomorphic forms on unitary groups, but there has been almost no interaction between these two disciplines.

Siegel Modular forms occur in the cohomology of local systems on moduli spaces of abelian varieties. Sometimes these moduli spaces are strongly related to moduli of curves. For example, for genus $\leq 3$ the moduli space of principally polarized abelian varieties is very close to the moduli space of curves. This fact and the fact that one can extract information about cohomology by using Frobenius over finite fields have been used very effectively to obtain a lot of new information about Siegel modular forms of genus $\leq 3$ and also for Picard modular forms. The link between the two topics that is thus obtained is an extremely useful tool. An example of an application is the disproval of the Gorenstein conjecture for the tautological ring of the moduli space $\mathcal{M}_{2,n}$ of $n$-pointed curves of genus 2.

Modular forms on ball quotients have not attracted much attention. Ball quotients are associated to moduli of abelian varieties associated to groups of type $U(n,1)$. But there are interesting links between various other moduli spaces in algebraic geometry and these Shimura varieties of type $U(n,1)$. For example, moduli of K3 surfaces and moduli of curves are linked in a number of papers by Kondo, Allcock and others to ball quotients and to modular forms on these ball quotients. The modular forms on ball quotients belong to the theory of automorphic forms on unitary groups, but there has been almost no interaction between the geometric aspects and the automorphic aspects.

Recently there has been a lot of activity on Kudla's program for unitary groups. Kudla and Rapoport defined special cycles on integral models of unitary Shimura varieties of type $GU(n,1)$ as the locus of abelian varieties (with additional data)

whose endomorphism ring contains certain special endomorphisms. They computed some of their arithmetic intersection numbers and related them to coefficients of derivatives of Eisenstein series. The height pairing of such Kudla-Raport divisors with CM cycles has been expressed as the derivative of the central value of a Rankin type $L$-function.

All these themes were well represented among the talks on this workshop. The great variety of topics treated became already visible on the first day. The workshop started with a beautiful survey talk of Farkas on his joint work with Alexeev, Donagi, Izadi and Ortega on the uniformization of the moduli space $\mathcal{A}_6$ of principally polarized abelian varieties of dimension 6. It was followed by a talk by S. Kondo who discussed Enriques surfaces in characteristic 2, an exceptional but intriguing case where the moduli space is reducible. He considered the question whether there exist Enriques surfaces in characteristic 2 with finite automorphism group and with a prescribed dual graph of the configuration of all smooth rational curves. He gave a 1-dimensional family of Enriques surfaces with a configuration of type VII constructed using Rudakov-Shafarevich derivations on K3 surfaces. The other two talks of the day showed the same diversity of topics. There was talk of Maryna Viazovska on her sensational work on the sphere packing problem: $E_8$ and the Leech lattice provide the densest possible sphere packings in dimensions 8 and 24. And Taïbi presented recent impressive advances on Siegel modular forms, obtained by using Arthur's multiplicity formula; he is able to derive explicit dimension formulae for spaces of vector-valued Siegel modular forms of level 1.

The diversity of the first day was continued on the following days as illustrated by the abstracts of the talks that follow hereafter. Some talks highlighted the algebraic geometry aspect of the topic, others concentrated on Arakelov geometry and there were talks dealing mostly with the modular forms side. The talks showed a wide range of topics but also presented quite a number of unexpected relations. The variation in the program was appreciated very much by the participants. It led to a very lively atmosphere with many discussions and a very fruitful workshop.

The organizers thank the staff of Oberwolfach for creating excellent working conditions during the workshop.

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## Workshop: Moduli spaces and Modular forms

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