

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Combinatorics and Probability

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ABSTRACT. For the past few decades, Combinatorics and Probability Theory have had a fruitful symbiosis, each benefitting from and influencing developments in the other. Thus to prove the existence of designs, probabilistic methods are used, algorithms to factorize integers need combinatorics and probability theory (in addition to number theory), and the study of random matrices needs combinatorics. In the workshop a great variety of topics exemplifying this interaction were considered, including problems concerning designs, Cayley graphs, additive number theory, multiplicative number theory, noise sensitivity, random graphs, extremal graphs and random matrices.

Mathematics Subject Classification (2010): 05xx, 60xx.

Introduction by the Organisers

The workshop was organized by Béla Bollobás (Cambridge and Memphis), Michael Krivelevich (Tel Aviv), Oliver Riordan (Oxford) and Emo Welzl (Zürich). The meeting was extremely well attended, with 53 participants from 13 countries, including the US, Israel, Canada, Australia, Brazil, Korea, and various European countries. Many excellent mathematicians who would have loved to participate could not be invited, for lack of space. The programme consisted of 11 main lectures, 14 shorter talks, and a problem session, with plenty of time for discussion.

The timing of the workshop was very fortunate, because recently several major results have been proved in probabilistic combinatorics and combinatorial probability. The main lectures provided a very good overview of these great results. In particular, Peter Keevash talked about his solution of a problem of Steiner from

1853 concerning the existence of designs, József Balogh, Wojciech Samotij and Andrew Thomason gave interconnected lectures on their extremely powerful *method of containers* and its applications, and Paul Balister talked about a sharp result on an old problem of Pomerance related to the fastest known algorithms for factoring large integers. In addition, Noga Alon presented a probabilistic result answering a basic extremal question concerning ‘universal’ graphs, David Gamarnik and Van Vu presented recent developments in the theory of random matrices, Asaf Shapira described a new version of Szemerédi’s regularity lemma (a fundamental tool in many areas of combinatorics and elsewhere), Mathias Schacht talked about a new class of extremal problems for hypergraphs, and Angelika Steger presented much simpler proofs of stronger forms of a number of results in randomized optimization. The shorter contributions, including several by younger researchers, covered a wide range topics. In the following we include the extended abstracts of all the talks, in the order they were given.

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