

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Diophantische Approximationen

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10 April – 16 April 2016

ABSTRACT. This number theoretic conference was focused on a broad variety of subjects in (or closely related to) Diophantine approximation, including the following: metric Diophantine approximation, Mahler's method in transcendence, geometry of numbers, theory of heights, arithmetic dynamics, function fields arithmetic.

Mathematics Subject Classification (2010): 11Jxx, 11K60.

Introduction by the Organisers

The workshop *Diophantische Approximationen* (Diophantine approximations), organised by Yann Bugeaud (Strasbourg), Philipp Habegger (Basel), and Umberto Zannier (Pisa) was held April 10th - April 16th, 2016. There have been 51 participants with broad geographic representation and a large variety of mathematical backgrounds. Young researchers were well represented, including among the speakers. Below we briefly recall the topics discussed, thus outlining some of the modern lines of investigation in Diophantine approximation and closely related areas. We refer the reader to the abstracts for more details.

Diophantine approximation is a branch of Number Theory that can be described as the study of the solvability of inequalities in integers, though this main theme of the subject is often unbelievably generalized. As an example, one can be interested in rational approximation to irrational numbers. Irrationality and transcendence have been discussed in the talks of Viola and Zudilin. Slightly related is the parametric geometry of numbers, recently introduced by W. Schmidt and Summerer, and which was at the heart of the talk of Roy.

Metric Diophantine approximation has seen great advances during the last decade and was present in the talks of Badziahin, Beresnevich, Haynes and Moshchevitin.

At the end of the 1920s, Kurt Mahler introduced the so-called *Mahler's method* to give new transcendence results. It has been recently revisited and several new advances were presented by Bell, Philippon and Zorin. A fourth related talk, on E - and G -functions was given by Rivoal.

Questions on algebraic number fields and on the theory of heights have been discussed by Widmer (with a connection to decidability questions), Amoroso, and Pottmeyer. Heights also played a role in Gaudron's talk. He presented explicit lower bounds for the Néron-Tate height on an abelian variety. Silverman discussed the vanishing locus of the Néron-Tate height attached to nef line bundles, a weakening of the ampleness criterion. Hindry presented evidence why the regulator and the Tate-Shafarevich group of an abelian variety are difficult to compute in the number and function field setting. Pazuki discussed the Northcott property for the regulator of an abelian variety. Kühne's talk was on a height bound for just likely intersections in semi-abelian varieties.

Bilu showed a strengthened estimate for the number of fields generated by fibers of rational functions over curves over integral values.

Bertrand spoke on Kummer theory for abelian varieties over a function field.

Functional transcendence in the spirit of the Ax-Schanuel Theorem played an important role in Corvaja's and Gao's talks. Corvaja discussed applications of Manin's Theorem of the Kernel and beyond to Pink's Conjecture on families of abelian varieties. Gao put the functional Schanuel Conjecture in a very general context and presented evidence in the case of a vectorial extension of an abelian variety. Harry Schmidt presented solvability results on the polynomial Pell equation.

Diophantine equations remain a subject of constant interest. Bennett combined a large variety of techniques to list integer squares with very few digits in some given base. Akhtari explained that many Thue equations have no solutions. Levin presented progress towards effective Shafarevich for genus 2 curves.

Various questions coming from ergodic theory and with a Diophantine flavour were discussed by Varju and Lindenstrauss.

Arithmetic dynamics is a quite recent topic of growing interest among the participants, and was the subject of the talks of Ingram, Tucker, and Zieve. Ingram exhibited a non-trivial family of non-Lattès maps that satisfy the Morton-Silverman uniformity conjecture. Tucker presented a dynamical analog of the Bugeaud-Corvaja-Zannier gcd bounds by taking composition of functions instead of powers of numbers. Zieve spoke on a variant of the Mordell-Lang Conjecture in the dynamical setting.

The abstracts are listed by order of appearance of the speakers during the conference.

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