

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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## Hochschild Cohomology in Algebra, Geometry, and Topology

Organised by

Luchezar L. Avramov, Lincoln  
Ragnar-Olaf Buchweitz, Toronto  
Wendy Lowen, Antwerpen

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**ABSTRACT.** In 1945 Gerhard Hochschild published *On the cohomology groups of an associative algebra* in the *Annals of Mathematics* and thereby created what is now called Hochschild theory. In 1963, Murray Gerstenhaber proved that the Hochschild cohomology of any associative algebra carries a super-Poisson algebra structure, comprised of a graded commutative cup product and an odd super Lie algebra structure that acts through graded derivations with respect to the product. Subsequently, a number of higher structures have been discovered, and a vast body of research concerning and/or using Hochschild theory has developed in many different fields in mathematics and physics.

*Mathematics Subject Classification (2010):* 16E, 13D, 14F, 55N, 83E30.

### Introduction by the Organisers

This meeting had 27 participants from 10 countries (Argentina[2], Belgium[3], Canada[2], China[3], France[4], Germany[1], Norway[3], Russia[2], UK[1], and the US[6]) and 20 lectures were presented during the five day period. The extended abstracts of these lectures are presented on the following pages in chronological order.

This workshop fostered exchange of knowledge and ideas between various research areas, developed existing collaborations, and identified new directions of research by bringing together leading researchers and young colleagues from Algebraic Geometry (in its classical and its noncommutative version), Singularity Theory, Representation Theory of Algebras, Commutative Algebra, and Algebraic

Topology. The choice of a coherent group of disciplines, rather than a broad coverage of Hochschild theory, allowed for effective communication between different groups of practitioners.

Survey lectures on Hochschild cohomology of algebraic varieties, the relationship between loop homology and Hochschild cohomology in algebraic topology, and on the Hochschild cohomology of block algebras of finite groups were complemented by presentations on higher order structures on Hochschild cohomology such as existence of a Batalin–Vilkovisky operator or the explicit form of the Gerstenhaber Lie bracket in special cases. Further, categorical interpretations of various aspects of Hochschild theory were presented, and variations of Hochschild cohomology such as Koszul or Poisson cohomology were studied.

Numerous discussions among the participants, in particular among participants belonging to different mathematical communities, have contributed to the workshop in an essential way. As always, such workshop at MFO provided an ideal atmosphere for fruitful interaction and exchange of ideas. It is a pleasure to thank the administration and the staff of the Oberwolfach Institute for their efficient support and hospitality.

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## Hochschild Cohomology in Algebra, Geometry, and Topology

### Table of Contents

Damien Calaque (joint with Carlo A. Rossi & Michel Van den Bergh)	
<i>Hochschild cohomology of smooth algebraic varieties</i> .....	453
Liyu Liu	
<i>Hochschild cohomology of projective hypersurfaces</i> .....	456
Dmitry Kaledin	
<i>Witt vectors as a polynomial functor</i> .....	456
Markus Linckelmann	
<i>On the Hochschild cohomology of finite group algebras</i> .....	458
Liran Shaul	
<i>Towards coherent duality over derived formal schemes</i> .....	462
Don Stanley	
<i>Loop homology and Hochschild cohomology</i> .....	465
Srikanth B. Iyengar (joint with Jon F. Carlson)	
<i>Tensor products with Carlson's <math>L_\zeta</math>-modules</i> .....	467
James Zhang	
<i>Auslander Theorem and Searching for Noncommutative McKay</i> .....	470
Alexander Zimmermann (joint with Bernt Tore Jensen, Xiuping Su; Manuel Saorín)	
<i>Degeneration in triangulated categories</i> .....	470
Sarah Witherspoon (joint with Lauren Grimley, Van C. Nguyen, Cris Negron)	
<i>An Alternate Approach to the Lie Bracket on Hochschild Cohomology</i> ..	473
Cris Negron (joint with Sarah Witherspoon)	
<i>The Gerstenhaber bracket as a Schouten bracket for polynomial rings     extended by finite groups</i> .....	476
Andrea Solotar (joint with Roland Berger, Thierry Lambre)	
<i>Koszul Calculus</i> .....	479
Yuri Volkov	
<i>Hochschild cohomology of a smash product with a cyclic group</i> .....	481
Zhengfang Wang	
<i>Singular Hochschild cohomology and Gerstenhaber algebra structure</i> ....	484

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Guodong Zhou	
<i>Batalin-Vilkovisky structures in Hochschild cohomology and Poisson</i>	
<i>cohomology</i> .....	486
Yang Han	
<i>Proper smooth local DG algebras are trivial</i> .....	490
Petter Andreas Bergh (joint with Magnus Hellstrøm-Finnsen)	
<i>Hochschild cohomology of ring objects</i> .....	492
Reiner Hermann (joint with Johan Steen)	
<i>The Lie bracket in Hochschild cohomology via the homotopy category of</i>	
<i>projective bimodules</i> .....	495
María Julia Redondo (joint with Lucrecia Román)	
<i>Hochschild cohomology of monomial algebras</i> .....	499
Travis Schedler	
<i>Quantizations of complete intersection surfaces and D-modules</i> .....	499