

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 5/2016

DOI: 10.4171/OWR/2016/5

Algebraic Cobordism and Projective Homogeneous Varieties

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31 January – 6 February 2016

ABSTRACT. The aim of this workshop was to bring together researchers in the theory of projective homogeneous varieties with researchers working on cohomology theories of algebraic varieties, so that the latter can learn about the needs in an area of successful applications of these abstract theories and the former can see the latest tools.

Mathematics Subject Classification (2010): 20Gxx, 14F42.

Introduction by the Organisers

Recent years have seen a new wave of results on quadratic forms and projective homogeneous varieties over semisimple algebraic groups. This development can be traced back at least in part to the introduction of methods from stable homotopy theory into the field, culminating in Voevodsky's proof of the Milnor conjectures and the proof of Rost and Voevodsky of the Bloch-Kato conjecture. As central as these two results are, the progress in the field is not limited to this. The introduction of new cohomological methods spawned by the push to prove the Milnor- and the Bloch-Kato conjectures has transformed the study of projective homogeneous varieties.

Much of this new impetus is due to the systematic extension of well-known cohomological techniques from algebraic topology, such as cohomology operations, to the setting of algebraic geometry. These tools are in turn descendants of the recent research in cohomology theories of algebraic varieties, including the universal oriented theory, algebraic cobordism, as well as a number of newly studied unoriented theories, such as hermitian K -theory, Witt theory and symplectic cobordism.

The aim of the workshop *Algebraic cobordism and projective homogeneous varieties* organized by Stefan Gille (Edmonton), Marc Levine (Essen), Ivan Panin (St. Petersburg), and Alexander Vishik (Nottingham) has been to bring together researchers in the theory of projective homogeneous varieties with researchers working on cohomology theories of algebraic varieties, so that the latter can learn about the needs in an area of successful applications of these abstract theories and the former can see the latest tools.

The workshop has been attended by about 50 researchers from Europe, North- and South-America and Asia, about 1/3 of them working on motives and/or \mathbb{A}^1 -homotopy theory, 1/3 on quadratic forms and related topics as algebraic groups and projective homogeneous varieties, and 1/3 in both of these areas. There have been 20 one hour talks. As was the intention of the workshop the organizers have taken some effort to keep the balance between talks which presented latest developments in motivic cohomology or \mathbb{A}^1 -homotopy theory, and talks which discussed recent applications to projective homogeneous varieties. New and strong results have been reported, which caused active discussion and interaction among the participants. New scientific connections and collaboration groups were formed.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”. Moreover, the MFO and the workshop organizers would like to thank the Simons Foundation for supporting David B. Leep in the “Simons Visiting Professors” program at the MFO.

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