

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 57/2015

DOI: 10.4171/OWR/2015/57

Non-Archimedean Geometry and Applications

Organised by
Vladimir Berkovich, Rehovot
Walter Gubler, Regensburg
Peter Schneider, Münster
Annette Werner, Frankfurt

13 December – 19 December 2015

ABSTRACT. The workshop focused on recent developments in non-Archimedean analytic geometry with various applications to other fields, in particular to number theory and algebraic geometry. These applications included Mirror Symmetry, the Langlands program, p-adic Hodge theory, tropical geometry, resolution of singularities and the geometry of moduli spaces. Much emphasis was put on making the list of talks to reflect this diversity, thereby fostering the mutual inspiration which comes from such interactions.

Mathematics Subject Classification (2010): 03 C 98, 11 G 20, 11 G 25, 14 F 20, 14 G 20, 14 G 22, 32 P 05.

Introduction by the Organisers

The workshop on Non-Archimedean Analytic Geometry and Applications, organized by Vladimir Berkovich (Rehovot), Walter Gubler (Regensburg), Peter Schneider (Münster) and Annette Werner (Frankfurt) had 53 participants. Non-Archimedean analytic geometry is a central area of arithmetic geometry. The first analytic spaces over fields with a non-Archimedean absolute value were introduced by John Tate and explored by many other mathematicians. They have found numerous applications to problems in number theory and algebraic geometry. In the 1990s, Vladimir Berkovich initiated a different approach to non-Archimedean analytic geometry, providing spaces with good topological properties which behave similarly as complex analytic spaces. Independently, Roland Huber developed a similar theory of adic spaces. Recently, Peter Scholze has introduced perfectoid

spaces as a ground breaking new tool to attack deep problems in p -adic Hodge theory and representation theory.

Recent years have seen a growing interest in such spaces since they have been used to solve several deep questions in arithmetic geometry. The goal of the workshop was to bring together researchers from different areas for an exchange of ideas which may facilitate future developments. Meanwhile, applications of non-Archimedean spaces have become so diverse that the workshop filled a gap in the recent list of conferences by providing a platform to exchange new results, ideas and open problems between the different branches of the subject. In fact, during the months before the workshop the organizers received numerous requests, also from some internationally renowned mathematicians, to be included in the list of participants.

We had 19 one hour talks in this workshop. A summary of the topics can be found below. All talks were followed by lively discussions, in the form of plenary questions and also in the form of blackboard discussions in smaller groups. Several participants explained work in progress or new conjectures or promising techniques to attack open conjectures. The workshop provided a lively platform to discuss these new idea with other experts.

During the workshop, we saw new structure results for affinoid spaces over the ring of integers (Poineau) and recent progress regarding skeleta of Berkovich spaces (Ducros and Loeser). Skeleta are polyhedral substructures which are deformation retracts, and which can be used to investigate the topology of Berkovich spaces. Loeser reported on his model-theoretic approach to skeleta (jointly with Hrushovsky) which leads to the proof of local contractibility of Berkovich spaces associated to varieties over non-Archimedean fields.

A surprising application of the non-Archimedean theory of skeleta to an important problem in diophantine geometry was presented in Rabinoff's talk. In joint work with Katz and Zureick-Brown, partial very explicit solutions of the uniform Mordell conjecture and of the uniform Manin-Mumford conjecture were proved. The power of non-Archimedean geometry to give classical problems a new point of view was also seen in Chambert-Loir's talk on a non-Archimedean Ax-Lindemann theorem and in Zhang's conjecture of a non-Archimedean Poisson formula.

Several talks dealt with progress in tropical geometry and tropical moduli spaces (Payne, Nicaise, Tyomkin, Ulirsch). Moreover, applications to mirror symmetry were presented, in particular regarding a new and very promising theory of intersections to deal with Gromov-Witten invariants (Yu). Geometric applications of non-Archimedean geometry for resolutions of singularities in positive characteristics are given via some precise analysis of de Jong's alterations (Temkin). Applications in positive characteristic included new results on p -adic curvature (Esnault).

A very influential recent development is Scholze's theory of perfectoid spaces, which is based on adic spaces and which has become a crucial tool in p -adic Hodge theory. In this area we have seen spectacular recent progress in a possible

reduction of the local Langlands program in number theory to a purely geometric analog of the geometric Langlands conjectures (Fargues). Related areas are relative p -adic Hodge theory (Kedlaya), p -adic representations (Strauch), Fourier transformations on \mathbb{Q}_p (Baldassari) and overconvergent modular forms (Hansen). Nizioł presented interesting results with Colmez for p -adic nearby cycles using syntomic cohomology.

Apart from the plenary talks, the participants had many discussions in small groups. The organizers made a specific effort to invite Phd students and Postdocs. Altogether we had 14 participants from this group. For most of them it was the first Oberwolfach workshop they ever attended. The unique Oberwolfach atmosphere provided a singular opportunity of meeting the international leaders of the subject and of keeping track with current developments. During the breaks and in the evenings many informal mathematical discussions took place, in which the young participants played an active role.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”. Moreover, the MFO and the workshop organizers would like to thank the Simons Foundation for supporting Ehud de Shalit in the “Simons Visiting Professors” program at the MFO.

Workshop: Non-Archimedean Geometry and Applications**Table of Contents**

Jérôme Poineau	
<i>Affinoid spaces over \mathbf{Z}</i>	3277
David Hansen (joint with Przemyslaw Chojecki, Christian Johansson)	
<i>Overconvergent modular forms: A perfectoid point of view</i>	3280
Michael Temkin	
<i>Desingularization by $\text{char}(X)$-alterations</i>	3282
Tony Yue Yu (joint with Mauro Porta)	
<i>Derived non-archimedean analytic spaces</i>	3284
Antoine Chambert-Loir (joint with François Loeser)	
<i>A non-archimedean Ax-Lindemann theorem</i>	3288
Hélène Esnault (joint with Mark Kisin)	
<i>p-curvature and connections</i>	3291
Shou-Wu Zhang	
<i>On non-archimedean Poisson's equation</i>	3292
Matthias Strauch (joint with Christine Huyghe, Deepam Patel, and Tobias Schmidt)	
<i>Arithmetic differential operators and representations of p-adic groups</i> ..	3295
Francesco Baldassarri	
<i>A p-adically entire function with integral values on \mathbb{Q}_p, Fourier transform of distributions, and automorphisms of the perfectoid open unit disc.</i> ...	3299
Kiran S. Kedlaya (joint with Ruochuan Liu)	
<i>Pseudocoherent sheaves and applications</i>	3302
Ilya Tyomkin	
<i>Algebraic-tropical correspondence for rational curves</i>	3303
Sam Payne (joint with Johannes Nicaise and Franziska Schroeter)	
<i>Refined curve counting, tropical geometry, and motivic Euler characteristics</i>	3305
Laurent Fargues	
<i>Geometrization of the local Langlands correspondence</i>	3307
François Loeser (joint with E. Hrushovski)	
<i>On skeleta</i>	3308
Johannes Nicaise (joint with Franziska Schroeter, Sam Payne)	
<i>Geometric invariants for non-archimedean semi-algebraic sets</i>	3310

Wiesława Nizioł (joint with Pierre Colmez)	
<i>Syntomic complexes and p-adic nearby cycles</i>	3312
Joseph Rabinoff (joint with Eric Katz, David Zureick-Brown)	
<i>Chabauty–Coleman on basic wide opens and applications to uniform boundedness</i>	3315
Martin Ulirsch	
<i>Logarithmic structures, Artin fans, and tropical compactifications</i>	3321
Antoine Ducros (joint with Amaury Thuillier)	
<i>Reified valuations spaces and skeletons of Berkovich spaces</i>	3323