

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mini-Workshop: Scales in Plasticity

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ABSTRACT. This mini-workshop was devoted to the current state of our understanding of dislocations (essentially slips of lines of atoms in a crystalline solid) and of their impact on the macroscopic behavior of those solids.

Mathematics Subject Classification (2010): Primary: 74 C15+C20, 49 J40+S05. Secondary: 35 J50, 74 K35+H35.

Introduction by the Organisers

The mini-workshop successfully brought together researchers with different expertise (experimentalists, engineers, theoretical mechanicians, applied mathematicians) working on plasticity. The main focus was on bridging the gap between the mesoscopic theory of what we would like to call the dislocation configuration and the macroscopic, mainly rate independent theory of plastic deformations. At the mesoscopic level, dislocations are viewed atomistically while the surrounding material is treated macroscopically as a purely elastic medium.

The range of topics that were discussed was extremely broad. In particular, the following topics were discussed:

- experimental results with accompanying detailed phenomenological laws;
- upscaling of dislocation walls;
- line tension models and iterated Gamma-convergence;
- differential geometric aspects of dislocations and disclinations;
- rate dependent dynamics of dislocations;
- gradient plasticity models and generalizations;
- dimensional reduction in small strain elasto-plasticity;

- plates with incompatible strains;
- structured deformations, among other themes.

Further, two discussion sessions were organized to confront viewpoints.

The input by the experimentalists in the group was extremely important. One of the outcomes of the discussion sessions was that there is still a large gap in our understanding of the “free energy” of neutral dislocation configurations, i.e., those with net Burgers vector zero. Another problem that emerged is the relationship between macroscopic hardening laws and the mesoscopic interaction of dislocations moving on a glide plane with the farthest dislocations transversal to the plane. In a different direction, the presentations in the workshop showed the variety of expertise in the application of energetic methods (Γ -convergence and energy descent flows) in the context of linear elasticity with defects, as well as in that of nonlinear elasticity with defects.

In the nonlinear elastic context, the workshop brought forth the need to develop the connection between the PDE approach to nonlinear elasticity and the differential geometric description of the elastic strain field à la Kondo and Kröner.

The topic of large scale dislocation dynamics simulation was unfortunately not represented at this workshop. This should be another topic for a future workshop or mini-workshop on plasticity.

Generally speaking, the participants felt that the mini-workshop format with less than twenty researchers was particularly successful in promoting discussions and new interactions. The week was pleasantly enhanced by the magnificent weather that was kind enough to stay with us even during the traditional walk to Saint Roman.

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