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## Scaling Limits in Models of Statistical Mechanics

Organised by  
Dmitry Ioffe, Haifa  
Gady Kozma, Rehovot  
Fabio Toninelli, Lyon

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ABSTRACT. The emphasis of the workshop was on the deep relations between, on the one hand, recent advances in probabilistic investigation of statistical mechanical models and spatial stochastic processes and, on the other hand, rigorous field-theoretic and analytic methods of mathematical physics. There were 52 participants, including 6 postdocs and graduate students, working in diverse intertwining areas of probability, statistical mechanics and field theory. Specific topics addressed during the 24 talks include: Universality and critical phenomena, disordered models, Gaussian free field (GFF), stochastic representation of classical and quantum-mechanical models and related random interchange and permutation processes, random planar graphs and unimodular planar maps, random walks on critical graphs and the Alexander-Orbach conjecture, reinforced random walks and non-linear  $\sigma$ -models, metastability, aging, equilibrium and dynamics for continuum particles with hard core interactions, non-equilibrium dynamics and Toom's interfaces.

*Mathematics Subject Classification (2010):* 60, 82.

### Introduction by the Organisers

The workshop was a sequel to three MFO conferences which took place in 2006, 2009 and 2012, and which were organized by Ken Alexander, Marek Biskup, Remco van der Hofstad and Vladas Sidoravicius. We tried to continue the tradition of organizing this series of workshops against the background of probabilistic and analytic methods of non-integrable statistical mechanics, this time with an emphasis on exchange of ideas between the experts in disjoint areas, specifically in rigorous Field Theory and in probabilistic Statistical Mechanics. The list of 52

invited participants reflects our attempts to maintain an optimal balance between diverse fields, leading experts and promising young researchers. Six participants were on postdoctoral and graduate level. One of the participants, Gordon Slade, was awarded Simons Visiting Professor fellowship.

In our choice of 24 talks we tried to illuminate major recent advances in the field and to expose and address at least some aspects of the works for each and every one of the participants. A more detailed account of the presentations is given below. Due to an intended intertwining of topics and themes it is hard to give an unambiguous classification.

*Universality, renormalization and critical phenomena.*

In a special session [Roland Bauerschmidt](#) explained profound ideas and techniques which were developed by Pierluigi Falco for an analysis of the Kosterlitz–Thouless transition line for the two-dimensional Coulomb gas.

[Gordon Slade](#) described recent results on criticality for four-dimensional weakly self-avoiding walks and  $\phi^4$  lattice fields via a rigorous renormalization group approach based on Berezin integration and the analysis of the flow of the effective coupling constants.

[Alessandro Giuliani](#) presented important results on: (1) universality of energy correlations and of free energy fluctuations for non-integrable two-dimensional Ising models and (2) universality of GFF-like height fluctuations of non-integrable two-dimensional dimer models.

[Michael Aizenman](#) explained how a random current representation leads to a proof of triviality of scaling theories in dimensions higher than four, and indicated how switching lemmas for the latter lead to fermionic Wick-type formulas in dimension two.

[Francesco Caravenna](#) presented results on universality of weak disorder limits, via chaos expansions, for a class of directed polymers, such as the  $(2 + 1)$  directed polymer or the  $(1 + 1)$  directed polymer with heavy tails.

A recent proof of the mean-field nature of critical percolation in dimensions larger than 10, via an enhanced method of lace expansions was outlined in the talk by [Remco van der Hofstadt](#).

[Asaf Nachmias](#) presented results about critical branching random walk in 5 dimensions, showing that some spectral exponents deviate from their mean-field values.

A proof of full RSW bound for crossing probabilities in the critical model of two-dimensional Voronoi percolation was explained in the talk by [Vincent Tassion](#).

Finally, [Wendelin Werner](#) gave an overview of Brownian loop soup and some new results connecting it to the gaussian free field.

*Quantum models, reinforced walks and permutations.*

A mixed random loop and random stirring representation for a class of quantum spin models was introduced by [Daniel Ueltschi](#) with a subsequent discussion of fascinating probabilistic interpretation of several open questions related to quantum phase transitions.

Shannon Starr explained his recent results on ordering of energy levels for quantum Heisenberg ferromagnet, and elucidated the relation with Aldous' spectral gap conjecture.

Emergence of microscopically large loops for random interchange process on the hyper-cube was discussed in the talk of Piotr Miłoś. Although the results fall short of proving Poisson-Dirichlet limiting statistics for macroscopic loop sizes, or even of proving existence of macroscopically large loops, an interesting fragmentation-coagulation structure was disclosed.

Ron Peled gave a different view on random permutations by discussing Mallows permutations model and band permutations.

Christophe Sabot presented results on the spectral properties of a random Schrödinger operator, which imply for instance recurrence of the Edge Reinforced Random Walk in dimension  $d = 2$  and a functional central limit theorem at weak disorder in dimension  $d \geq 3$ .

Margherita Disertori surveyed relations between the vertex-reinforced jump process, the edge-reinforced random walk and the supersymmetric hyperbolic sigma model.

*Discrete Gaussian free field (GFF), spin waves and related models.*

Hubert Lacoin gave a lecture on the  $d$ -dimensional GFF interacting with a flat interface via a disordered pinning potential. For  $d > 2$ , the critical point of the pinning transition coincides with that of the annealed model, while the critical exponent is modified by quenched disorder. This contrasts the situation for  $d = 1$ .

Thomas Richthammer presented results in the challenging field of continuous particle systems. His results quantify the absence of breaking of translation invariance in two-dimensional hard disk models: the variance of a particle's position w.r.t. its ideal crystalline position is at least the logarithm of the system size.

Marek Biskup presented deep results which fully describe, in terms of a decorated Poisson-Dirichlet process, the extremal process of the two-dimensional lattice GFF and its conformal invariance properties.

*Random planar graphs and unimodular planar maps.*

Omer Angel showed a new characterization of hyperbolic planar triangulations using a notion of discrete curvature.

Nicolas Curien presented universality results for random planar maps, showing that, using various natural definitions of distance on these graphs, large balls grow in the same way, up to a constant multiplicative factor.

*Random walks, long time behavior of equilibrium and non-equilibrium dynamics, metastability and aging.*

Jiří Černý presented recent results on aging for the Metropolis dynamics of the Random Energy Model: aging is proved without the usual non-physical assumption that the process is a time change of the simple random walk on the hypercube.

Sabine Jansen gave a lecture on metastability phenomena for a Metropolis dynamics of continuous particles, and presented results on nucleation time and shape of critical droplets.

Nicholas Crawford discussed several results related to Toom's model, a nonequilibrium particle system.

Balint Toth presented a superdiffusive central limit theorem, valid in any dimension, for the displacement of a test particle in the periodic Lorentz gas in the limit of large times  $t$  and low scatterer densities (Boltzmann-Grad limit).

*Summary.* The workshop was an obvious success. In particular, it helped to update the participants on the state of the art and on the important pending open problems in the fields related to their domain of research, facilitated exchange of ideas between researchers in technically disconnected areas, and it gave rise to many interesting and informative discussions, which were conducted either during 10 minutes discussion time allocated after each and every 50 minute talk, or during afternoon breaks or during the evenings, all of which were kept free.

*Acknowledgement:* The organizers would like to thank the MFO personnel for the help and for the invaluable logistic support, as well as for creating a friendly and stimulating environment throughout the entire meeting.

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