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Subfactors and Conformal Field Theory

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Abstract. Connections between subfactor theory and conformal field theory have been expected since the early days of the former in 1980's, and recently we see more and more evidence for deeper relations. It was our aim to attract experts from a wide range of topics related to subfactors and CFT. Many of the participants met for the first time at Oberwolfach, and there were numerous very fruitful interactions.

Mathematics Subject Classification (2010): 17B69, 18D10, 46L37, 46L60, 47L90, 81R15, 81T05, 81T40.

Introduction by the Organisers

Subfactor theory in today's form was initiated by Jones in the early 1980's. It has revolutionized the theory of operator algebras and through it, many surprising connections to low-dimensional topology, quantum groups, statistical mechanics and quantum field theory were discovered. Two-dimensional conformal field theory has been well studied during the last 30 years, and it has also been connected to vast ranges of subjects in mathematics and physics. Formal similarities between subfactor theory and conformal field theory were apparent since the early days, but we have recently seen more and more explicit connections. The workshop gathered mathematicians and physicists covering a wide range of topics in these and related areas.

(1) Subfactors and fusion categories

The Haagerup subfactor was found as an exceptional ("exotic") subfactor in 1990's. Its siblings and generalizations have been studied by many researchers, but

their real meaning has not been understood. The Haagerup subfactor has been believed to be connected to conformal field theory and vertex operator algebras, particularly through the work of Evans-Gannon, but still many important details have to be clarified. Also the representation theoretic aspects of subfactor theory have recently caught much attention.

We had talks of V. Jones, Evans, Morrison, Snyder, C. Jones, Vaes, Izumi, Grossman, Penneys, Brothier, Liu and Shlyakhtenko on these topics. Haagerup was among the participants. We also had an informal talk of Gannon on an approach from subfactors to non-unitary fusion categories on Thursday evening.

(2) Algebraic quantum field theory

In algebraic quantum field theory, we study nets of observable algebras parameterized by spacetime regions. This approach has found deep connections to subfactor theory in the 1980s through works of Longo and Fredenhagen-Rehren-Schroer. Two-dimensional conformal field theory has been extensively studied in this context and the associated mathematical object is called a local conformal net. Its connection to the theory of vertex operator algebras, algebraic axiomatizations of chiral conformal field theory was not well-understood beyond many apparent formal similarities, but Carpi gave a talk on connecting the two theories directly for the first time. Longo, Tanimoto, Rehren, Müger and Bischoff also gave talks on these topics.

(3) Vertex operator algebras

A vertex operator algebra first appeared in studying Monstrous Moonshine in 1980's through works of Borcherds and Frenkel-Lepowsky-Meurman. VOAs were well developed to a sophisticated theory over many years. Though it was started independently from algebraic quantum field theory and has had deep connections to algebra such as finite sporadic simple groups and modular functions, it has become clear that VOAs must be closely related to algebraic quantum field theory. There are many similarities between the two theories. Mason, Duncan and Lam gave talks on these topics.

(4) Conformal field theory and tensor categories

Various aspects of conformal field theory have been studied in the context of modular tensor categories. Unitary tensor categories have been extensively studied, but non-unitary ones also appear, particularly in connection to logarithmic conformal field theory. Schweigert, Runkel, Fuchs, Creutzig and Schommer-Pries gave talks on these topics.

(5) Other topics

Tener gave a talk on an example of a Segal type conformal field theory. Teichner talked about moduli spaces of field theories, and Henriques gave a talk on Stolz-Teichner cocycles. Wassermann's talk was about analysis on trinions. Ogata explained some of her work on gapped Hamiltonians in quantum statistical mechanics.

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