

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Algebraic Geometry

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ABSTRACT. The workshop covered a broad variety of areas in algebraic geometry and was the occasion to report on recent advances and works in progress. Special emphasis was put on the role of derived categories and various stability concepts for sheaves, varieties, complexes, etc. The mix of people working in areas like classification theory, mirror symmetry, derived categories, moduli spaces, p -adic geometry, characteristic p methods, singularity theory led to stimulating discussions.

Mathematics Subject Classification (2010): 14Cxx, 14Dxx, 14Exx, 14Fxx, 14Gxx, 14Jxx, 32Qxx, 32Sxx.

Introduction by the Organisers

Algebraic geometry is a vast and thriving subject with a countless number of researchers in the field worldwide. While most conferences focus on more specialized topics, this workshop was designed to give a broader view on various aspects of algebraic geometry with the aim to spread ideas across subfields. To make this really happen we targeted researchers with a broad range of interests, working on topics that usually require a mix of different techniques. The result was an intense exchange of ideas, with a very attentive and lively audience throughout the 21 talks of 50 minutes each, and continuing with productive discussions in the lunch breaks and after dinner, often late into the evening. The schedule was sufficiently relaxed to permit free time and to consent to recover energy for discussions. This worked out perfectly well and the atmosphere was generally felt to be extremely stimulating and productive. In fact, many participants expressed

their satisfaction about the design of the workshop and the new mix of topics and people. The fact that many young participants got the chance to present their work was generally very appreciated.

The big success of the workshop was due to the high quality of the participants, with a large number of prime players together with many young but already very visible participants. The level of the workshop is also illustrated by the fact that half of the participants came from overseas.

A larger number of talks was devoted to derived categories of coherent sheaves, addressing questions of semi-orthogonal decompositions for a particularly interesting class of Fano manifolds (Alexander Kuznetsov), non-commutative enhancements and deformations related to rational curves in Calabi-Yau varieties (Will Donovan), spaces of stability conditions on abelian and Calabi-Yau varieties (Arend Bayer), a conceptually new approach to stability of objects (Daniel Halpern-Leistner), derived categories of moduli spaces under wall crossing (Matthew Ballard) and moduli spaces of stable objects in derived categories (Yukinobu Toda).

New results on moduli spaces of sheaves on surfaces were presented in talks by Giulia Saccà. In a joint work with Arbarello, she describes the singularities of the moduli space of sheaves on K3 surfaces in strictly semistable points in terms of quiver varieties, which is important for the understanding of wall crossing phenomena. Aspects of mirror symmetry were highlighted in the talks by Helge Ruddat (mirror symmetry for conifold transitions) and Alessio Corti (classification of Fano surfaces via their mirror Landau-Ginzburg potentials).

Talks by János Kollár (Numerical flatness and stability criteria), Zolt Patakfalvi (Projectivity of moduli spaces of KSBA stable pairs) and Chenyang Xu (Degeneration of Fano Kähler-Einstein manifolds) concentrated more on foundational problems related to moduli theory of algebraic varieties.

In her talk, Enrica Floris explained a recent result with Paolo Cascini addressing deformation invariance of plurigenera for foliations of surfaces. This is in analogy to Siu's result, one of the central results in classification theory, but the case of foliations turns out to be considerably more involved.

The talk by Bhargav Bhatt reported on ongoing work with Peter Scholze that transports a well-known result on the structure of the affine Grassmannian as an ind-projective scheme in characteristic zero to the p -adic case. Besides its fundamental importance, the role of Keel's criteria for basepoint freeness in positive characteristic made this of particular interest to participants working in classification theory.

Rationality questions have always been of special interest to algebraic geometers. In his talk Burt Totaro reported on very recent results on hypersurfaces of not too small degree not being stably rational, which strengthens earlier results by Kollár proving non-rationality.

François Charles explained his new approach to the Tate conjecture for K3 surfaces which is based on a version of Zarhin's trick for K3 surfaces via moduli spaces of stable sheaves and which uses boundedness results for birational equivalence classes of hyperkähler manifolds. The method eventually shows finiteness

of (unpolarized) K3 surfaces over finite fields which had been shown to imply the Tate conjecture.

Although not giving talks themselves, the presence of more senior participants like Paolo Cascini, Gerard van der Geer, Ludmil Katzarkov, Jun Li, Mircea Mustata, Mihnea Popa, and Karl Schwede was important for the success of the workshop. There was a lively exchange of ideas between the generations which was appreciated by all.

The Mathematische Forschungsinstitut Oberwolfach provided an excellent environment and inspiring atmosphere for this workshop and we are grateful for its hospitality.

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