

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Combinatorial Optimization

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ABSTRACT. Combinatorial Optimization is an area of mathematics that thrives from a continual influx of new questions and problems from practice. Attacking these problems has required the development and combination of ideas and techniques from different mathematical areas including graph theory, matroids and combinatorics, convex and nonlinear optimization, discrete and convex geometry, algebraic and topological methods. We continued a tradition of triannual Oberwolfach workshops, bringing together the best international researchers with younger talent to discover new connections with a particular emphasis on emerging breakthrough areas.

Mathematics Subject Classification (2010): 90C27 (Combinatorial Optimization); secondary: 90C57, 90C10, 90C11, 90C22, 90C06, 90C59, 90C90.

Introduction by the Organisers

There has been a tradition of triannual Oberwolfach workshops in Combinatorial Optimization and the 2014 edition was again a great success.

The Oberwolfach workshops have truly played a key role for our field, which cannot be substituted by any other workshop or conference. This success is due to Oberwolfach's reputation for excellence, and its outstanding research conditions as well as the unique format of the workshop.

As in the past, the program consisted of five one-hour focus lectures planned in advance (one on each day of the workshop), and shorter presentations scheduled during the workshop. As in the past, we made sure to leave sufficient time for discussions and research in small groups. We also continued the last workshop's

successful micro-presentations (5 minutes, strictly timed) so that all participants had an opportunity to present their hottest recent result or burning open question.

The focus lectures covered topics where recent progress has been most intense, and future progress seems very promising:

Geometric Approach to Cutting Planes (Michele Conforti)

Semidefinite Extended Formulations (Rekha Thomas)

Algebraic Geometry (Jean Lasserre)

Lattice Algorithms (Damien Stehlé)

Complexity of the Union of Polyhedra (Juan Pablo Vielma)

During the workshop, we were excited to see great advances in several new directions. The synergy between discrete and continuous models is emerging as a thriving area of importance, and new techniques from pure mathematics continue to transform combinatorial optimization, such as the use of tools from *algebraic geometry*. This is the case for instance in the theory of *integer programming*, which is under rapid development using techniques from several fields, such as *lattice algorithms* and *semidefinite (SDP) bounds*. There was an exciting advance in this area just prior to the workshop due to Lee-Raghavendra-Steurer: a first super-polynomial lower bound on the complexity of SDP projections yielding certain combinatorial polytopes (such as the travelling salesman polytope). The solution was presented at the workshop by James Lee. Finally matroids and structural *graph theory* have been at the heart of many important advances in combinatorial optimization. We were fortunate to hear from Gyula Pap about one of two independent (and distinct) solutions (the second due to Satoru Iwata) that has been announced to the longstanding open problem of determining whether weighted linear matroid matching is polytime solvable.

We would like to thank all participants for their carefully prepared contributions and the many exciting discussions. Last but certainly not least, we thank the Oberwolfach Research Institute and its members for providing the outstanding meeting and working conditions and the unique inspiring Oberwolfach atmosphere.

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Workshop: Combinatorial Optimization

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