Abstract. Locally compact groups are ubiquitous in the study of many continuous or discrete structures across geometry, analysis and algebra. Every locally compact group is an extension of a connected group by a totally disconnected group. The connected case has been studied in depth, notably using Lie theory, a culminating point being reached in the 1950s with the solution to Hilbert’s 5th problem. The totally disconnected case, by contrast, remains full of challenging questions. A series of new results has been obtained in the last twenty years, and today the activity in this area is witnessing a sharp increase. These texts report on the recent Arbeitsgemeinschaft on this topic.

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Introduction by the Organisers

Locally compact groups arise as the symmetry groups of all sorts of structures across many areas of mathematics. This includes Lie groups, $p$-adic and adèlic groups, isometry groups of general proper metric spaces. Even discrete structures such as locally finite graphs give rise to very interesting locally compact automorphism groups. Besides the groups themselves, one of the most important motivations to study locally compact groups is that they frequently appear as the “envelope” in which abstract groups of interest appear as lattices. This is notably the case for arithmetic groups and Kac–Moody groups. It has often happened that the most interesting theorems about those abstract groups are proved by transferring the problem to the ambient locally compact group and solving it there.
In the study of locally compact groups, it is usually understood that the focus is on non-discrete groups since otherwise it remains within “abstract” group theory. The case of Lie groups has been extensively studied for well over a century and largely classified in the early twentieth century. The next significant period of research culminated in the 1950s with the solution to Hilbert’s Fifth Problem, giving a satisfactory picture of the connected case.

Therefore, the main locus of modern research on locally compact groups is the study of non-discrete totally disconnected locally compact groups, since a general locally compact group decomposes as an extension of a connected group by a totally disconnected group.

The revival of this topic can arguably be dated to the work of G. Willis starting two decades ago. This gave a new impetus to the study of the local structure of totally disconnected groups. More recently, there has been progress both on the global and local structure. In addition, the compact case (i.e. profinite groups) has also witnessed important recent progress on the algebraic side.

The goals of the Arbeitsgemeinschaft are: to learn the necessary prerequisites, to study substantial parts of the recent developments and to reach the point where open problems can be discussed.

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## Arbeitsgemeinschaft: Totally Disconnected Groups

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