

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 42/2014

DOI: 10.4171/OWR/2014/42

Topologie

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14 September – 20 September 2014

ABSTRACT. The Oberwolfach conference “Topologie” is one of only a few opportunities for researchers from many different areas in algebraic and geometric topology to meet and exchange ideas. The program covered new developments in fields such as automorphisms of manifolds, applications of algebraic topology to differential geometry, quantum field theories, combinatorial methods in low-dimensional topology, abstract and applied homotopy theory and applications of L^2 -cohomology. We heard about new results describing the cohomology of the automorphism spaces of some smooth manifolds, progress on spaces of positive scalar curvature metrics, a variant of the Segal conjecture without completion, advances in classifying topological quantum field theories, and a new undecidability result in combinatorial group theory, to mention some examples. As a special attraction, the conference featured a series of three talks by Dani Wise on the combinatorics of $CAT(0)$ -cube complexes and applications to 3-manifold topology.

Mathematics Subject Classification (2010): 55-xx, 57-xx, 18Axx, 18Bxx.

Introduction by the Organisers

This conference was the third topology conference in Oberwolfach organized by Thomas Schick, Peter Teichner, Nathalie Wahl and Michael Weiss. About 50 mathematicians participated, working in many different areas of algebraic and geometric topology.

The talks were of three types. There were 14 regular one-hour talks, 3 one-hour talks by keynote speaker Dani Wise and a “gong show” where 12 young speakers had the opportunity to present their research in 10 minutes each, including question time.

The 15 regular talks of the conference covered a wide range of topics such as spaces of automorphisms of highly connected manifolds, spaces of Riemannian metrics with positive scalar curvature, new developments in abstract homotopy theory, techniques for solving equations in groups, undecidability results in combinatorial group theory, algorithms in 3-manifold topology, a variant of the (affirmed) Segal conjecture which does away with the need for finite completion, and new developments in topological and other quantum field theories. Speakers were instructed to give talks that could be appreciated by an audience of topologists of many different kinds, and they were generally very successful in doing so.

Keynote speaker Dani Wise spoke on $CAT(0)$ -cube complexes, his work in the theory and how it became an essential ingredient in the recent spectacular proof of the virtual Haken conjecture by Ian Agol (Berkeley). He concentrated on the combinatorial aspects, giving a very patient introduction to the geometric properties of $CAT(0)$ -cube complexes in the first two talks and sketching applications to 3-manifold topology in the last one. His vigorous delivery made these talks as riveting as we could have wished.

The gong show with 12 speakers took place on Wednesday morning. In the opinion of this writer, it is a hard training for the young, but there is no doubt at all that the speakers rose to the occasion. Dieter Degrijse and Irakli Patchkoria both talked on their joint work relating the virtual cohomological dimension of groups G to the homotopy theory of what they call proper G -spectra. Lukasz Grabowski spoke on his work in the theory of L^2 invariants to disprove the conjecture of Lott and Lück that the Novikov-Shubin invariants are always positive. Holger Kammeyer reported on a proof of the Farrell-Jones conjecture in algebraic K - and L -theory for arbitrary lattices in connected Lie groups. Christina Pagliantini presented a new result on Gromov’s simplicial volume for hyperbolic 3-manifolds. Daniel Kasprowski spoke on the Farrell-Jones conjecture in algebraic K -theory for groups with finite decomposition complexity. Daniela Egas Santander offered a comparison of various combinatorial models of moduli spaces of two-dimensional cobordisms and some compactifications, relating for example the graph models of Godin and Costello to Bökigheimer’s model designed along more classical lines. Pedro Boavida de Brito talked about his work in functor calculus, spaces of smooth embeddings and operad theory with applications to spaces of higher-dimensional long knots. Both Daniel Tubbenhauer and Lukas Lewark spoke on developments in the Khovanov homology of knots, Tubbenhauer more on relations with representation theory and Lewark more on applications to genus-type invariants of slice knots. Markus Upmeyer spoke on a theorem of his establishing the existence of a moment map for certain actions of symplectomorphism groups. Finally Nat Stapleton reported on a new proof and generalization of a result of Strickland’s regarding some generalized cohomology of symmetric groups.

We now describe the themes of the regular 1-hour talks.

Oscar Randal-Williams talked about his joint work with Galatius (Stanford) on the cohomology and related invariants of spaces of automorphisms of smooth manifolds. One point of departure for this, years ago, was the affirmed Mumford conjecture on the cohomology H^* of spaces of automorphisms of surfaces of genus g where $g \gg *$. But the current level of generality allows for manifolds of dimension $2n$, where $n \neq 2$, having the form of a connected sum of a fixed and rather arbitrary smooth manifold with g copies of $S^n \times S^n$.

Category theory teaches us, as soon as we have learned to reason with sets and elements, that we should not reason quite so much with elements. In Emily Riehl's talk about formal category theory we learned the next lesson: do not reason quite so much with objects and morphisms. To begin with she described axioms/conditions isolating key features of the category of categories as a 2-category. More desirable features were added to the framework as the talk went on. In a similar vein, Ieke Moerdijk talked about categories for homotopy theorists, specifically about categories of functors from a small category \mathcal{A} to spaces, and a comparison of that, for homotopy theorists, with the category of spaces over the classifying space $B\mathcal{A}$.

Wolfgang Lück spoke on L^2 -torsion invariants and relations between these and the Thurston norm on the first cohomology of irreducible 3-manifolds.

Johannes Ebert spoke about new results on spaces of positive scalar curvature metrics. The proofs rely on the results of Randal-Williams and Galatius on automorphisms of some smooth manifolds, but also on steady progress in the theory of surgery on manifolds with a positive scalar curvature metric.

Both Owen Gwilliam and Chris Schommer-Priess talked on aspects of quantum field theories. Schommer-Priess talked about progress related to the Stolz-Teichner program, a conjectural parameterization of some elementary quantum field theories. Gwilliam's talk was on the deformation quantization of a type of classical field theory, emphasizing the cohomological meaning of existence and (non-)uniqueness of such deformation quantizations and describing some associated computations. The talk by Zsuzsanna Dancso appeared to have some intriguing connections with the mathematics of quantum field theory though she did not say so. Her theme, abstractly stated, was homomorphic expansions of planar algebras. She explained how in the case of a particular planar algebra, obtained from a topological setup, these homomorphic expansions are equivalent to solutions of an important equation in Lie theory, and how they can be constructed using the topological setup.

Andrew Putman spoke on homological stability phenomena with dimension shift in the cohomology of $SL_n\mathbb{Z}$. His talk had strong connections to number theory.

Gerd Laures' talk was a survey of recent computations of the generalized cohomology (closely related to TMF) of important classifying spaces such as $BSpin$ and $BString$.

The Segal conjecture, affirmed in the mid 1980s, states that (a form of) the 0-th stable cohomotopy group of the classifying space of a finite group G is isomorphic to the Burnside ring of G completed at the augmentation ideal. It is one of the

great triumphs of stable homotopy theory. In Jesper Grodal's talk we heard about a variant of the Segal conjecture which involves the uncompleted Burnside ring.

Quite a few talks were related to algorithms or solvability statements of use in algebraic topology. Martin Bridson spoke on the non-decidability of the existence of finite-index subgroups in finitely presented groups. Andreas Thom's talk was on methods for solving certain equations in countable groups. Saul Schleimer gave a survey of algorithms and algorithmic problems in 3-manifold topology.

Once again the Oberwolfach staff, not least the kitchen staff, helped to make this meeting pleasant and memorable. Our thanks go to the institute for creating this atmosphere and making the conference possible.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, "US Junior Oberwolfach Fellows". Moreover, the MFO and the workshop organizers would like to thank the Simons Foundation for supporting Craig Westerland in the "Simons Visiting Professors" program at the MFO.

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