Abstract. One of the fundamental problems in Riemannian geometry is to understand the relation of locally defined curvature invariants and global properties of smooth manifolds. This workshop was centered around the investigation of scalar curvature, addressing questions in global analysis, geometric topology, relativity and minimal surface theory.


Introduction by the Organisers

The workshop Analysis, Geometry and Topology of Positive Scalar Curvature Metrics was attended by some fifty participants from Europe, the US, South America and Japan, including a number of young scientists on a doctoral or postdoctoral level. Rather than representing a single mathematical discipline the workshop aimed at bringing together researchers from different areas, but working on similar questions. Hence special emphasis was put on the exchange of ideas and methods from mathematical physics, global analysis and topology, providing an attractive and diverse scientific program.

The foundations of the subject were laid in the sixties, seventies and eighties in the work of Kazdan-Warner, Lichnerowicz, Hitchin, Schoen-Yau and Witten, among others: On the one hand it is well known that each smooth function on a closed smooth manifold $M$ of dimension at least 3 can be realized as the scalar curvature of a Riemannian metric on $M$, if and only if $M$ admits a metric of positive scalar curvature. On the other hand, by combining the Weitzenböck formula
from spin geometry with the Atiyah-Singer index theorem, a closed spin manifold with non-vanishing $\hat{A}$-genus cannot carry a metric of positive scalar curvature. In combination with the Kazdan-Warner result, which relies on the analysis of geometric PDEs, this reveals a deep interplay of the theory of geometric PDEs and subtle differential-topological invariants. Additional obstructions based on minimal hypersurfaces and the positive mass theorem establish close connections to variational methods and general relativity.

All of these, by now classical, aspects play important roles in today's research and were represented in the scientific activities during the workshop. After two ninety minutes survey lectures on index theory and general relativity the state of the art was unfolded in sixteen one hour lectures and eight short contributions.

Among the broad range of subjects a major theme was the discussion of the Einstein constraint equations from general relativity including the optimal localisation of asymptotically flat metrics of non-negative scalar curvature on Euclidean space.

A number of talks was devoted to invariants detecting the topology of the space of positive scalar curvature metrics on a fixed manifold based on higher $\rho$-invariants and up to date methods from differential topology.

The discussion of the Schoen-Yau minimal hypersurface obstruction for non-smooth metrics of positive scalar curvature, of the indices of minimal surfaces and of large scale obstructions to positive scalar curvature metrics like macroscopic dimension were the content of three more talks.

Further topics of interest were the construction of Riemannian metrics satisfying certain criticality conditions on connected sums of Einstein manifolds and the discussion of the Yamabe invariant on products of manifolds.

The long time properties of Ricci solitons, metrics with invertible Dirac operators on spin manifolds and their behaviour under surgery, spectral properties of the conformal Laplacian and metrics of almost non-negative sectional curvature also attracted special attention.

Shorter contributions by young participants dealt with equivariant aspects of the positive scalar curvature problem, its relation to stable homotopy theory and higher index theory, and the index theory of pseudodifferential operators on open manifolds, among others.

An informal problem session at the end of the conference collected important topics for future investigations.

The interdisciplinary character of the workshop was reflected by the fact that the lectures were not on a highly specialized, technical level, but accessible to an audience with different mathematical backgrounds.

The atmosphere during the workshop was both relaxed and inspiring, with many questions and discussions during and outside the lectures. A perfect organization and management by the staff of the Oberwolfach institute created an optimal working environment.
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