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Stochastic Analysis: Around the KPZ Universality Class

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Abstract. The Gaussian distribution is the "universal" distribution arising in a huge variety of contexts that describes the compound effect of the random fluctuations of many independent (or weakly dependent) sources of randomness that are combined in a (close to) additive way. While this has been very well understood for a long time, the last few years have seen an explosion of results around the "KPZ universality class", which contains many systems where strongly interacting individual components are combined in a highly non-linear way. In this class, which is still rather poorly understood from a mathematical perspective, fluctuations typically exhibit scaling exponent 1/3 instead of the exponent 1/2 familiar from the central limit theorem and limiting distributions are of Tracy-Widom type rather than Gaussian.

This workshop brought together outstanding researchers from a variety of mathematical backgrounds whose areas of research are linked to the understanding of the KPZ equation and universality class. While there are strong links between their motivations, the techniques used by these researchers span a large swath of mathematics, ranging from purely algebraic techniques to renormalisation theory, stochastic analysis, random matrix theory, classical probability theory, orthogonal polynomials, the theory of rough paths, etc.

Mathematics Subject Classification (2010): 60xx, 35xx, 82xx.

Introduction by the Organisers

The workshop focused on the latest progresses in the study of the Khardar-Parisi-Zhang (KPZ) equation and its universality classes. In recent years, new important models were shown to belong to this KPZ class thanks in particular to integrable

systems and orthogonal polynomials, approaches to universality florished, in particular in random matrix theory, and the rigorous construction of the solution to KPZ equation was achieved thanks to Malliavin calculus and rough paths analysis.

Most of the conference was devoted to the study of exact models. In the first talk, H. Widom revisited his breakthrough paper with C. Tracy and discussed the analysis of the asymmetric simple exclusion process and how to transform a priori untractable formulas by wise uses of complex analysis and changes of contours of integration. A. Borodin surveyed the range of known exact models in the KPZ class which could be solved, and emphasized the role of Fourier analysis and orthogonal polynomials, in particular those introduced by Mac Donald.

Later in the week, I. Corwin showed in great detail how one of these models, the q-TASEP, could be analyzed either by using MacDonald processes or a new duality approach, both leading to the exact computation of natural observables. V. Gorin talked about new applications of these techniques to the six-vertex model. P. Ferrari showed on the last day how the study of these exact models, and in particular the so-called q-Whittaker process, allowed to get an exact formula for the one point correlation function of the solution to the KPZ equation. In the same vein, A. Hammond described a new approach to KPZ equation as the lowest indexed curve of an N-ensemble of curves, with a property to penalize, but not completely forbid ding crossing: this ensemble is integrable and offers a powerful tool for the analysis of KPZ equation. P. Le Doussal explain the physics approach to these questions by replicas and Bethe ansatz. It was discussed by D. Remenik that putting some of the results discussed by Le Doussal on a firm mathematical ground can be a real challenge, for instance for the model of random growth off a flat surface where justifying some formal critical point analysis is yet to be done. S. Chiita and K. Johansson discussed the two-periodic weighting of the Aztec diamond: its asymptotics were described by Kenyon and Okounkov and the authors undertake the analysis of the fluctuations and the two points correlations functions of this model in the different phases, by exhibiting new exact formulas. Event though pairwise correlations at the solid-liquid and liquid-gas region can be analyzed, the liquid-gas phase remains a challenge due to non-local phenomenon. Other models could be analyzed and shown to be related to the KPZ equation. This is the case of weakly interacting particle systems presented by P. Gonçalves, or directed polymers as shown by T. Seppäläinen. Other universal behaviors also appear for the latter model, as discussed by K. Khanin.

During the first morning, random matrices and universality were introduced by P. Bourgade, who showed how recent techniques based on Dyson Brownian motion and homogenization could be used to prove fixed energy universality of the correlation function. A. Knowles discussed the phase diagram for the mesoscopic phase diagram of d-dimensional random band matrices, the appearance of the Altshuler-Shklovskii and its universality. B. Virág described the Dirac operator, a random first order differential operator, and showed that the local statistics of its spectrum in the bulk converges to the same limit as those of β-ensemble. For $\beta = 2$, the eigenvalues are conjectured to behave similarly to the imaginary

parts of the zeroes of the Riemann ζ-function. B. Valkó presented matrix analogue to Dufresne identities. O. Zaboronski showed that Pfaffian point processes, which usually appear in random matrix models, make a surprise appearance when studying annihilating Brownian motions in one dimension.

On topics related to the well-posedness and interpretation of the KPZ equation itself, P. Friz gave an introduction to rough path analysis and stressed their importance to define and study the KPZ equation. M. Gubinelli described the slightly different approach based on paracontrolled analysis. D. Khoshnevisan discussed stochastic differential equation with a constant or linear diffusivity, presenting intermittency and multi-fractality, and Y. Bakhtin exposed a recent result on the uniqueness of stationary solutions to the forced Burgers equation on the whole real line which is conjectured to belong to the KPZ universality class.

A special occasion, very nice and enjoyable, was also the Oberwolfach prize ceremony that took place on Thursday late afternoon, and was followed by a nice dinner (and special thanks go to the Oberwolfach staff that made us all feel being in a luxury restaurant on that evening). The winner of the prize, Hugo Duminil-Copin who was also a participant to this workshop, gave a talk on the nature of the phase transition of the three-dimensional Ising model (joint work with Aizenmann and Sidoravicius), which is a topic close to the heart of many participants of this workshop (hence also the choice of the week of this workshop to have this ceremony).

The role of the organizers was to keep the number of talks to a fairly low number (22 all together) in order to leave as much time as possible for informal sessions and discussions between participants, and to look at the weather forecast to advance the traditional mid-week hike (this time to Brandenkopf) to Tuesday afternoon instead of the indeed very rainy Wednesday.

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