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Mini-Workshop: Batalin-Vilkovisky Algebras, Operads, and Hopf Algebroids

Organised by
Vladimir Dotsenko, Dublin
Ulrich Krähmer, Glasgow

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Abstract. This workshop brought together 17 researchers whose work involves Batalin–Vilkovisky algebras, operads, and related structures such as Gerstenhaber algebras and cyclic homology. It featured introductory lectures on some relevant topics followed by talks about recent research results.

Mathematics Subject Classification (2010): 18D10, 18D50, 18G30, 19D55, 55P48, 55P50, 83C47.

Introduction by the Organisers

A lot of research in homological algebra comes in one of two flavours: some subjects of study have their origins in algebraic topology (model categories, simplicial objects, operads etc), the others in algebra and algebraic geometry (Tor and Ext, derived categories). The two are strongly interacting, and problems and solutions have often found their way from one community into the other. But there is still some cultural difference, and the wider aim of this workshop was to bring researchers from both backgrounds together. Batalin-Vilkovisky algebras have cropped up recently in various places in both algebra and algebraic topology. Thus, we have decided to make these the central topic of the workshop, as they were an ideal catalyst for discussions.

The two other topics of the workshop are promising directions for interaction, topics in which the two communities can learn from one another. Operads, originally invented by topologists, have been used for the purposes of homotopical algebra and deformation theory in a prominent way in the past two decades, but still are finding their place in the repertoire of methods of contemporary algebra.

Hopf algebroids undeservedly have not yet attracted full attention of experts in operad theory, and this workshop appeared a perfect opportunity to fix that.

In short, a Batalin-Vilkovisky (BV from now on) algebra is a graded commutative algebra V equipped with a differential Δ. This differential is not assumed to turn V into a differential graded algebra. Instead, the failure of the (graded) Leibniz rule,

$$\{v, w\} := \Delta(vw) - \Delta(v)w - (-1)^{\deg v} v\Delta(w), \quad v, w \in V,$$

is assumed to turn V into a Gerstenhaber algebra. Thus BV algebras are a special type of Gerstenhaber algebra in which the bracket is generated by a differential; hence they are alternatively referred to as exact Gerstenhaber algebras.

Historically, the notion was coined in the BRST formalism in quantum field theory but more recently it has become clear that cohomology rings of various mathematical objects tend to have a canonical BV algebra structure. For example, the paradigmatic example of a Gerstenhaber algebra is the Hochschild cohomology of an associative algebra, and this is BV whenever the algebra is a Calabi-Yau algebra [4]. An analogous result holds for Lie-Rinehart cohomology [5] and in fact the Ext-algebra of any Hopf algebroid [6]. In the operadic world, BV algebras arise form operads with multiplication [8].

In these references, Poincaré duality identifies the cohomology of the BV operator with a suitable variation of cyclic homology. Furthermore, Koszul-type dualities relate the BV operator in some cases to Frobenius algebra structures on Koszul dual objects [10]. However, this is so far only established for concrete cohomology theories and examples, the deeper reason behind these mechanisms seems not yet fully understood.

The original application in quantum field theory provides a homological interpretation of some physical equations. From a mathematical point of view, this can be seen as a special case of homological perturbation theory, or homotopy transfer formulae [9].

In a slightly different context, BV algebras have been applied in symplectic topology, see e.g. [1], and one of the aims of the workshop was to make the algebraic audience aware of these results.

However, the general aim was to report on the most up-to-date developments that the participants want to share. Hence we asked a few speakers to give introductory and survey lectures that explained some central notions and results, and afterwards the remining speakers gave research talks on whatever topic they felt was most relevant to the workshop.

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Table of Contents