Mathematisches Forschungsinstitut Oberwolfach

Report No. 01/2014

DOI: 10.4171/OWR/2014/01

Combinatorics

Organised by
Jeff Kahn, Piscataway
Angelika Steger, Zürich
Benny Sudakov, Los Angeles

5 January – 11 January 2014

Abstract. Combinatorics is a fundamental mathematical discipline which focuses on the study of discrete objects and their properties. The current workshop brought together researchers from diverse fields such as Extremal and Probabilistic Combinatorics, Discrete Geometry, Graph theory, Combinatorial Optimization and Algebraic Combinatorics for a fruitful interaction. New results, methods and developments and future challenges were discussed. This is a report on the meeting containing abstracts of the presentations and a summary of the problem session.

Mathematics Subject Classification (2010): 05xx.

Introduction by the Organisers

The workshop Combinatorics organized by Jeff Kahn (Piscataway), Angelika Steger (Zürich), and Benny Sudakov (Zürich) was held January 5th - January 11th, 2014. Despite the early point in the year the meeting was extremely well attended with roughly 50 participants from the US, Canada, Australia, UK, Israel, and various European countries. The program consisted of 13 plenary lectures, accompanied by 17 shorter contributions and the special session of the presentations by Oberwolfach Leibniz graduate students. There was also a vivid problem session led by Andrew Thomason. The plenary lectures were intended to provide overviews of the state of the art in various combinatorial areas and/or in-depth treatments of major new results. The short talks ranged over a wide variety of topics including graph theory, coding theory, discrete geometry, extremal combinatorics, Ramsey theory, theoretical computer science, and probabilistic combinatorics. Special attention was paid throughout to providing a platform for younger researchers to present themselves and their results.

This report contains extended abstracts of the talks and the statements of the problems that were posed during the problem session. This was a particularly successful edition of the meeting Combinatorics, in large part because of the exceptional strength and range of the results discussed. Here we mention just two of these, each of which involved spectacular progress on some well-known, longstanding conjecture. These two snapshots also provide a nice, if small, sample of the large variety of topics and methodologies that were presented during a fascinating week in Oberwolfach. We add in passing that we deliberating did not offer a long talk for one of the most spectacular – and far reaching – results in combinatorics, proved just two years ago: the so called hypergraph container theorem. While the results (and the authors!) certainly would have deserved this, we felt we would bore a large part of the audience (who likely heard reports on hypergraph containers already several times) by adding a formal talk. Thus, this result was just present at lunch and dinner tables and, indirectly, in various small talks.

Now back to the two talks that we would like to explain in a little detail. A Steiner system with parameters (n, q, r) is a set S of q-subsets of an n-set X, such that every r-subset of X belongs to exactly one element of S. The question of whether there is a Steiner system with given parameters is one of the oldest problems in combinatorics, dating back to work of Plücker (1835), Kirkman (1846) and Steiner (1853). More generally, we say that a set S of q-subsets of an n-set X is a *design* with parameters (n, q, r, λ) if every r-subset of X belongs to exactly λ elements of S.

There are some obvious necessary 'divisibility conditions' for the existence of such S, namely that $\binom{q-i}{r-i}$ divides $\lambda\binom{n-i}{r-i}$ for every $0 \leq i \leq r-1$ (fix any i-subset I of X and consider the sets in S that contain I). It is a very old Existence conjecture that the divisibility conditions are also sufficient, apart from a finite number of exceptional n given fixed q, r and λ. The case $r = 2$ has received particular attention because of its connections to statistics, under the name of 'balanced incomplete block designs'. The Existence Conjecture even for $r = 2$ was a long-standing open problem, eventually resolved by Wilson in 70's in a series of papers that revolutionized Design Theory. In his talk at the workshop Peter Keevash announced a proof of the Existence Conjecture in general, via a new method, which can be called Randomized Algebraic Constructions.

In 1912 George Birkhoff introduced a function $\chi_q(G)$, defined for all positive integers q and finite graphs G, which counts the number of proper colorings of G with q colors. It turns out that $\chi_q(G)$ is a polynomial of q with integer coefficients. This polynomial is called the chromatic polynomial of G. It appears that coefficients of the chromatic polynomial have many intriguing properties. One such property was conjectured more than 40 years ago by Read. A sequence of real numbers $a_0, a_1, \ldots, a_n$ is called *log*-concave if for all $0 < i < n$,

$$a_{i-1}a_{i+1} \leq a_i^2.$$

In 1968 Read conjectured that if $\chi_q(G) = a_n q^n - a_{n-1}q^{n-1} + \cdots + (-1)^n a_0$ is the chromatic polynomial of the graph G, then the sequence $a_0, a_1, \ldots, a_n$ is *log-concave*. At the workshop, June Huh discussed how tools from Algebraic Geometry can be used to attack such problems and presented a proof of Read's conjecture.

On behalf of all participants, the organizers would like to thank the staff and the director of the Mathematisches Forschungsinstitut Oberwolfach for providing such a stimulating and inspiring atmosphere.

Workshop: Combinatorics

Table of Contents