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Mini-Workshop: New Crossroads between Mathematics and Field Theory

Organised by
Romeo Brunetti, Trento
Christian Bär, Potsdam
Claudio Dappiaggi, Pavia
Klaus Fredenhagen, Hamburg

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Abstract. In the last few years, it has been strongly emphasized the need to use new mathematical tools and structures which are not part of the traditional pool of expertise of the community working on the analysis of the mathematical and structural properties of classical and quantum field theory. Goal of the workshop has been to bring together some of the major experts in these topics to discuss the latest results and the new insights brought to field theory by techniques, such as microlocal analysis, infinite dimensional geometry and homological algebra.

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Introduction by the Organisers

Goal of the workshop was, on the one hand, to bring together some of the major experts in the analysis of the mathematical aspects of classical and quantum field theory and, on the other hand, to discuss the key open problems as well as the main mathematical tools which are expected to play an active role in future researches. The hope was not only to look for opening new collaborations, but also to discuss new potential approaches to the various questions in this field, which are still unanswered. The event has been attended by 17 participants and it started with two introductory lectures, the first by Klaus Fredenhagen on the present status of algebraic quantum field theory and the second by Chris Fewster on the concept of Hadamard states and on the related quantum energy inequalities.

The most represented community in this event works on the algebraic approach to quantum field theory (AQFT). This is a well-established branch of mathematical physics which emphasizes the role of observables and their interplay with the notion of locality and causality. From a physical point of view, it has the net advantage of being essentially the only procedure which can be applied naturally when the underlying background is curved and thus it leads to concrete applications to cosmology. Overall it can be described as a two-step procedure. In the first one, a unital $*$-algebra of observables is assigned to a specific physical system. In this framework there have been several leaps forward during the past few years and, in particular, the formulation of all free field theories on arbitrary globally hyperbolic spacetimes is a topic fully understood. This particular aspect and, more precisely, the construction of the algebra of observables for a large class of free fields as well as their quantization via canonical commutation or anticommutation relations has been discussed in detail by Christian Bär during his talk.

The second step in the algebraic approach to quantum field theory is the assignment of a state, that is a normalized linear functional from the algebra of observables to the complex numbers which fulfills a positivity condition. Unfortunately not all states are physically acceptable and a precise mathematical characterization of those which are admissible has to be provided. It is universally accepted that the correct answer to this query are the so-called Hadamard states which are defined in terms of the wavefront set of their associated truncated two-point function. While the existence of such states is guaranteed by an old result by Wald, Fulling and Narcovich, their explicit construction is still an open issue which has been thoroughly investigated during the workshop. In his talk Christian Gérard has shown that a possible way to tackle this problem for a scalar field originates from a careful use of pseudo-differential calculus. Another approach has been discussed by Claudio Dappiaggi and it goes under the name of bulk-to-boundary correspondence. In spacetimes possessing a null (conformal) boundary it is possible to construct an injective $*$-homomorphism between the algebra of observables of the theory under analysis and an auxiliary one living intrinsically on the boundary. As a byproduct every state on the boundary induces a counterpart in the bulk and there exists a distinguished use of this procedure which allows to impose also the Hadamard condition. The biggest drawback of this method is the necessity to know in detail the fall-off condition of the solutions of the equation of motion for the field under analysis in the limit when they approach the null boundary and future timelike infinity. For this reason there is a close connection between these methods and the novel techniques aimed at the study of the stability of black holes. These were thoroughly discussed by Mihalis Dafermos in his talk.

Another open issue, which has been discussed extensively during this mini-workshop, is the analysis in the algebraic approach of gauge theories. Several frameworks have been proposed and outlined: For Abelian structure groups, one can exploit the affine character of the bundle of connections in order to quantize Yang-Mills theories adapting the procedure used for the usual standard linear fields. Alexander Schenkel reported on this topic and he has also emphasized that an explicit characterization of the full gauge group is possible for the Abelian scenario. As a by-product one is able to identify and classify all observables which probe Aharonov-Bohm configurations. A connected problem consists of the construction of Hadamard states for such class of theories. In comparison with standard linear field theories, one has to cope with the additional difficulty of encoding gauge invariance. As reported by Alexander Strohmaier for the case of a $U(1)$ vector potential, it is possible to generalize also to curved backgrounds the Gupta-Bleuler mechanism, used in Minkowski spacetime. Hence existence of Hadamard states for a $U(1)$ Yang Mills theory is guaranteed.

For non-Abelian structure groups and more general scenarios, a possible approach consists of employing the Batalin-Vilkovisky formalism as reported by Katarzyna Rejzner. Although this approach has the net advantage of working in great generality, it leads naturally to deal with infinite dimensional spaces and differential calculus thereon. This is a topic, which is still intensively studied in pure mathematics and its applications to algebraic quantum field theory require careful analyses, particularly in connection with the microlocal properties inherited by the underlying Hadamard states. This issue has been discussed by Christian Brouder in this talk.

The use of infinite dimensional calculus within the framework of gauge theories comes parallel to an investigation on the possibility to discuss non linear classical field theories from an algebraic point view and in the spirit of the principle of general local covariance. This challenging investigation has been at the heart of the talks of Romeo Brunetti and of Pedro Laurdisen Ribeiro who introduced, most notably, the principle of spacetime descent, as a novel way to describe internal symmetries in classical field theories. The expectation is that a formulation also at the quantum level might be possible.

During the mini-workshop, concrete applications of the latest results in algebraic quantum field theory to physical phenomena and models have also been discussed. In particular Valter Moretti has shown how it is possible to translate into a rigorous framework the argument of Parikh and Wilczek which describes Hawking radiation as a tunneling process through the event horizon of a black hole. Most notably, in the talk, it has been shown how this procedure can be made intrinsically local and how the result holds true also for a polynomially interacting scalar field, at least at first order in the perturbation series. Applications to cosmology have been reported in the talks of Thomas-Paul Hack and Nicola Pinamonti. The former has discussed perturbations in inflationary models, seen as the linearization of a coupled Einstein-Klein-Gordon system. Thomas-Paul Hack has emphasized the role of gauge invariance in particular and he has discussed the related construction of the algebra of observables, also in view of the recent results in algebraic quantum field theory on the treatment of gauge systems. Nicola Pinamonti has discussed, instead, the use of semiclassical Einstein's equations in a cosmological framework under the further assumption that the matter content of the Universe can be described via a massive, conformally coupled scalar field. On the one hand he has shown via a fixed point theorem argument that global solutions for this highly coupled system exist. On the other hand he has reinterpreted semiclassical Einstein's equations as an equation for the probability distributions of the curvature and of the the matter stress-energy tensor. In this procedure he has followed the same procedure according to which Brownian motion is treated as arising from a Langevin equation.

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