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Mini-Workshop: The Willmore Functional and the Willmore Conjecture

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Abstract. The Willmore functional evaluated on a surface immersed into Euclidean space is given by the L^2-norm of its mean curvature. The interest for studying this functional comes from various directions. First, it arises in applications from biology and physics, where it is used to model surface tension in the Helfrich model for bilipid layers, or in General Relativity where it appears in Hawking's quasi-local mass. Second, the mathematical properties justify consideration of the Willmore functional in its own right. The Willmore functional is one of the most natural extrinsic curvature functionals for immersions. Its critical points solve a fourth order Euler-Lagrange equation, which has all minimal surfaces as solutions.

Mathematics Subject Classification (2010): 53C42, 58E12, 35J48, 35J60, 49Q10.

Introduction by the Organisers

In recent years there has been substantial progress concerning analytical and geometrical questions related to the Willmore functional. Highlights include the study of surfaces with square integrable second fundamental form, the compactness of $W^{2,2}$-conformal immersions, the regularity of weak solutions of the Willmore equation and the resolution of the longstanding Willmore conjecture.

The aim of this mini-workshop was to bring together people involved in propelling the above mentioned research highlights. In particular, our intention was to make a connection between the experts on minimal surfaces and corresponding min-max techniques that were a crucial ingredient in the proof of the Willmore conjecture, and the experts for the analysis developed for second order curvature functionals such as the Willmore functional.

For this purpose two mini-courses were delivered by Fernando Marques (Min-max theory and the Willmore conjecture) and Tristan Rivière (The variations of the Willmore Lagrangian, a parametric approach). Moreover, every participant gave a talk, with plenty of time left for discussions.

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