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Extremes in Branching Random Walk and Branching Brownian Motion

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Abstract. Branching random walk (BRW) and branching Brownian motion (BBM) are mathematical models for population growth and spatial displacement. When resources are plentiful, population sizes grow exponentially in time. In such a situation, exceptional (or *extreme*) individuals will be found far from the bulk of the population. The study of such individuals, and their ancestral lineages, was the subject of the workshop. On one hand, this is a classical topic, with well-known connections to the KPP-equation and to search algorithms. On the other hand, substantial recent developments have recently been obtained via new approaches to the subject (stopping lines and spines, the view from the tip, multivariate analytic combinatorics), or from researchers working in seemingly distinct areas (from stochastic partial differential equations to theoretical physics).

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Introduction by the Organisers

A *branching random walk* (BRW) is a system of particles in some physical space such as $\mathbb{R}^d$, where, on the one hand, particles reproduce independently of one another according to some fixed distribution, and on the other, the displacement of particles with respect to their parent's position are also independent from one another, and are distributed according to some fixed law. *Branching Brownian motion* is also a process which involves branching and spatial displacement, but with particle trajectories given by Brownian motions.

Branching random walks and branching Brownian motion are fundamental objects of interest in probability, and have been studied in some depth at least since the 1950s. Yet many basic aspects of their behaviour remain poorly understood to this date – this is particularly true for questions regarding the extremal behaviour of such processes, which were the main focus of this workshop. While these are natural and intrinsic questions from a mathematical standpoint, it has appeared in recent years that they are also of fundamental importance in other scientific fields, from the analysis of search algorithms in computer science to the understanding of energy landscapes in random energy and spin glass models from theoretical physics, and including the description of the effect of natural selection on the genealogy of populations in theoretical biology.

There has been recently a surge of new ideas and breakthroughs which are moving the subject closer to the resolution of some of its longstanding problems. Strikingly, some of these key advances have been made simultaneously and independently by different groups of researchers, usually through entirely different methods. We outline a few themes which featured prominently.

– *Minimal position.* Bramson, Ding and Zeitouni; Addario-Berry and Reed; and Aïdékon have all proved related results on the position of the minimum and on its convergence in distribution; these results build on existing work by Biggins, Devroye, and McDiarmid, among others. These recent results are established using very different techniques. For instance, Aïdékon relies on spine methods and derivative martingales. Bramson, Ding and Zeitouni relied on robust versions of second moment arguments in a way which allowed them to treat the case of the discrete two-dimensional Gaussian Free Field. Addario-Berry and Reed use a rather combinatorial argument based on the second moment method. Some recent developments in this direction include the works of Arguin, Bovier and Kistler; and Aidékon, Berestycki, Brunet and Shi who show the existence of a limit for the point process of particles near the minimum for a branching Brownian motion. Moreover, this process is ergodic if correctly recentered by the derivative martingale (in order to take into account the early fluctuations of the process).

– *Aldous' conjecture.* This conjecture concerns the following situation: assume that all particles that reach a certain subset of $\mathbb{R}^d$ (say the negative half-line on $\mathbb{R}$) are immediately killed and removed from the system. Then there is a critical value β_c for the branching rate such that if $\beta \leq \beta_c$, the system dies out with probability 1, while it survives with positive probability if $\beta > \beta_c$. At $\beta = \beta_c$, Aldous conjectured that (in one dimension) the total number of individuals Z satisfies $\mathbf{E}(Z) < \infty$ but $\mathbf{E}(Z \log Z) = \infty$. Addario-Berry and Broutin made an even stronger conjecture that $\mathbf{P}(Z > x) \sim c/(x \log^2 x)$. Variants of this conjecture have been independently established by several researchers: by Maillard for branching Brownian motion, who introduced new ideas and substantially developed the singularity analysis approach; by Aidékon, Hu and Zindy, based on a trajectorial decomposition; and by Aidékon via spine methods.

– *Genealogies.* When a population evolves by branching and with a selection mechanism that maintains a fixed population size, only letting the fittest (extreme) particles survive, physicists Brunet and Derrida have made striking, non-rigorous predictions for the genealogy of the resulting population. Most notably, they predicted a characteristic "genealogical timescale" of $(\log N)^3$ generations if N is the population size, and a genealogy described by the Bolthausen-Sznitman coalescent. Aspects of this conjecture have recently been established by Bérard and Gouéré and by Berestycki, Berestycki and Schweinsberg, all matching perfectly the predictions. Relation to noisy travelling waves and in particular the stochastic KPP equation (which underpinned the non-rigorous approach of Brunet and Derrida) remain mysterious, even though separate but related predictions which they made for the propagation of the wavefront in this equation were recently proved by Mueller, Mytnik and Quastel. The Bolthausen-Sznitman coalescent is also the conjectured limiting object for the energy landscape in random energy models and related spin glass models. This is not a coincidence, but most aspects of this connection remain to be understood.