Mathematisches Forschungsinstitut Oberwolfach

Report No. 12/2013

DOI: 10.4171/OWR/2013/12

From "Mixed" to "Applied" Mathematics: Tracing an important dimension of mathematics and its history

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3 March – 9 March 2013

Abstract. The workshop investigated historical variations of the ways in which historically boundaries were drawn between 'pure' mathematics on the one hand and 'mixed' or 'applied' mathematics on the other from about 1500 until today. It brought together historians and philosophers of mathematics as well as several mathematicians working on applications. Emphasis was laid upon the clarification of the relation between the historical use and the historiographical usefulness and philosophical soundness of the various categories.

Mathematics Subject Classification (2010): 01A40 to 01A99.

Introduction by the Organisers

For a long time, research in the history of mathematics has mostly focused on developments in pure mathematics. In recent years, however, some significant research has changed this situation. The conference brought together historians of mathematics actively involved in this reorientation, in order to take stock of what has been achieved, and to identify historical problems yet to be solved. In having done so, the organizers hope to have advanced the general understanding of the relations of mathematics with neighbouring scientific fields, and with technological, economic and social practices involving mathematical methods, in the period since ca. 1500.

A fundamental historical insight provided a starting point for planning this meeting: There is no, and there has never been, a once and for all fixed notion of 'applied' mathematics. Rather, we have to deal with a field of interactions

of the production of mathematical knowledge with a large and variable number of scientific, technological and social areas beyond the core disciplines of 'pure' mathematics. For want of a better term, and without taking the term literally, we call this field the 'applied field' of mathematics.

At different times in history, conflicting views on the value and possibilities of using mathematical methods in other contexts have generated controversial discussions e.g. about the unity of mathematics, about the role of mathematics as a tool, or as an aim in itself. Motives for engaging in such debates came both from the side of the practitioners of the mathematical sciences themselves and from the side of real or potential 'users' of mathematical methods. Moreover, the very notion of the 'application' of ready-made mathematical methods and knowledge to extra-mathematical domains is problematic; in fact in many cases mathematical methods emerged from interactions with such domains, thereby changing and challenging the existing ideas about mathematics.

In the early modern period, the mathematical sciences in Europe were formed with reference to an inner distinction between *mathematicae purae* and *mathematicae mixtae.* However, the field denoted as 'mixed mathematics' was complex, varying in inner structure, and not coinciding with what later came to be called 'applied' mathematics (a denotation whose meaning was hardly less variable). The situation was further complicated by the fact that mathematical methods became part of the very notion of 'natural philosophy', or science per se, during the 17th century. Several sciences – and first and foremost the new science of 'mechanics' in all its branches from celestial mechanics to hydrodynamics – were inextricably linked with the production of mathematical knowledge. During the 18th century, the notion of 'application' of one mathematical science to another was gaining interest (e.g. in the approach of the French Encyclopedists, in particular of Jean d'Alembert). Moreover, in several fields of the mathematical sciences (such as hydrodynamics and hydraulics), a lack of mutual integration of 'theory' and 'practice' was lamented by several authors at the time. Nevertheless, as is clearly shown in the work of Euler and many of his contemporaries, there was still no delineated field of 'applied mathematics'. A more explicit distinction between 'pure' and 'applied' mathematics gradually came to the fore during the 19th century, while an institutional separation of 'applied' and 'pure' mathematical research in journals, university positions and, eventually, institutes was a matter of the 20th century. During the early 20th century, moreover, the idea of 'mathematical modelling' was shaped, and it rapidly gained importance in a wide variety of areas from economics to engineering and medicine. The revolutionary development of new calculation devices contributed to a new balance between analytical, graphical and numerical methods and led finally even to a new notion of the solvability of mathematical problems. These developments, initially taking place primarily within applied contexts, also became important for certain areas of pure mathematics. In recent decades, however, voices are gaining strength that argue for a renewed integration of pure and applied mathematics, both on the institutional and cognitive level (such voices are already well-known from the beginning of the 20th century when

Richard Courant was one of their strongest speakers). Striking a balance between the development of the applied field and the unity of mathematics remains a challenge.

In view of the above, the scope of topics dealt with during the conference needed to span the period between the 16th century and the present, in order to help us better understand how the 'applied field' developed and was structured. Throughout, the emphasis was not to establish a separate historiography of applied mathematics per se, but rather to understand how the whole of mathematics was internally structured both with a view to 'applications' and to its relation with extra-mathematical domains.

By giving short introductory talks the three organizers tried to raise some general issues and questions for the ensuing 20 presentations and the discussion. These talks went into the notion of applied mathematics originating around 1800, into historical changes in the classification of sub-branches of mathematics according to the pure/mixed or pure/applied divide (with particular reference to influential works such as the French *Encyclopédie* and comparing them with modern changes in the relationship between science and technology), and, thirdly, the very important features of modeling within modern applied mathematics, the latter with emphasis on models in economics and biology.

While biology as an area of application was followed up in the conference in a talk on Karl Pearson, last minute cancellations resulted in a reduced discussion of applications in economics. However, the latter topic came up in connection with a presentation of social statistics in Late Imperial China. This contribution was also exceptional with respect to its non-European focus. At the same time the two last named presentations touched upon statistics as an important field and tool of applied mathematics, as did other talks on applied mathematics in industrial surroundings, which in particular discussed applied work by Iris Runge and B.L. van der Waerden. Engineering mathematics was recognized in the workshop as an important historical bridge and stimulus for renewing the relationship between mathematics and the applied field since the 18th century. Related to this topic were talks on mathematizing water powered machines and on ballistics in the 18th and 19th centuries and another presentation on the important hybrid discipline of fluid dynamics around 1900. The important theme of visualization of mathematical concepts and its changes from material models of the pre-computer age (used for instance by Felix Klein) to modern computer imaging was touched upon in several talks. Mathematical physics, which had often been a topic of historical discussions before, was less completely represented in the workshop although it came up particularly in connection with talks discussing the development of French, German and English applied mathematics during the 19th century. In this context regional and national differences between the various mathematical cultures, which partly still exist in modern globalized research, came clearly to the fore.

The workshop aimed from the outset at emphasizing the role of the applied field in the period of early modern mathematics. This was done in presentations on

hitherto little known areas such as 'mathematical gardening' or landscape design, and it was related to mathematical practices in early modern treatises on mathematical perspective and the development of mathematical instruments during the same period. The aspect of auxiliary instruments and means for computation and construction in applied mathematics and the development of the material infrastructure were discussed in various contributions in connection with the treatment of observational data, with mathematical tables, handbooks and textbooks of mathematics. This discussion reached as far as touching the topic of computer design at the hands of John von Neumann in its relation to theoretical work by Alan Turing. However, similar to mathematical physics, the development of computing technology – a topic frequently treated in other contexts and occasions – was not a focus of the workshop.

Throughout the workshop problems of conceptual development were discussed, e.g. the relation between (astronomical) observation and mathematical theory, the discussion of 'inner-mathematical' applications in the case, for instance, of Hermite, the conflict between the factual and indubitable use of notions such as 'mixed' and 'applied' mathematics and the philosophical and logical insufficiencies of the latter.

Thus both in philosophical and conceptual respects and as to the possible fields and directions of application (economy, non-European cultures, mathematical physics, mathematical instruments and computers) there is room for enlarging the future discussion of the history of mathematics in the applied field.

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