Mathematisches Forschungsinstitut Oberwolfach

Report No. 07/2013

DOI: 10.4171/OWR/2013/07

Mini-Workshop: Numerical Upscaling for Media with Deterministic and Stochastic Heterogeneity

Organised by
Yalchin Efendiev, College Station
Oleg Iliev, Kaiserslautern
Panayot Vassilevski, Livermore

10 February – 16 February 2013

Abstract. This minisymposium was third in series of similar events, after two very successful meetings in 2005 and 2009. The aim was to provide a forum for an extensive discussion on the theoretical aspects and on the areas of application and validity of numerical upscaling approaches for heterogeneous problems with deterministic and stochastic coefficients. The intensive discussions during the meeting contributed to a better understanding of upscaling approaches for multiscale problems with stochastic coefficients, and for synergy between scientists coming to this topic from the area of deterministic multiscale problems on one hand, and those coming from the area of SPDE on the other hand. Recent advanced results on upscaling approaches for deterministic multiscale problems were presented, well mixed with strong presentations on SDE and SPDE. The open problems in these areas were discussed, with emphasis on the case of stochastic coefficients brainstorming numerous numerical upscaling approaches. A number of young researchers, very actively working in these areas, were involved in the workshop discussing the links between scales., thus ensuring the continuity between the generations of researchers.

Mathematics Subject Classification (2010): 65, 68, 70, 74, 76.

Introduction by the Organisers

1. Aim and Motivation

This minisymposium was third in series of similar events. The previous two minisymposia on Numerical Upscaling took place in 2005 and 2009 and were very successful. In particular, the last one, in 2009, concentrated on differences and

similarities of FE/FV based numerical methods for multiscale problems from one side, and algebraic multigrid methods for the same problems, from another side. The intensive discussions on the common points between these two approaches allowed identifying the topics for synergetic developments. Further on, there was a second core of the discussions there, and this was the synergy between developing method for upscaling multiscale problems, and developing robust multilevel preconditioners for multiscale problems.

The aim of the third mini-workshop, held in 2013, was to provide a forum for an extensive discussion on the theoretical aspects and on the areas of application and validity of numerical upscaling approaches for heterogeneous problems with deterministic and stochastic coefficients. The term "numerical upscaling" is used here to denote several approaches (to be discussed below) for studying multiscale problems in the case when the scales cannot be separated (e.g., heterogeneity exist at each scale and the asymptotic homogenization is not directly applicable). The uncertainty discussed, concerns stochastic elliptic PDEs and systems of PDEs. A recapitulation of the achievements on upscaling approaches for deterministic multiscale problems was done, accompanied by intensive discussion on the open problems in the area. Strong presentations on SDE and SPDE allowed to better understand the specifics of these problems. Brainstorming on numerical upscaling approaches for the case of stochastic coefficients was among the main goals of the meeting.

2. Background

Multiscale problems, due to their importance for many branches of science and industry, attract significant attention of the mathematical community. Essential success was achieved during the last decades in the studies of problems with clearly separated fine and coarse scales (e.g., periodic microstructures, which are heterogeneous at a fine scale, but are homogeneous at a coarse scale). In this case the ratio between the fine and the coarse scale can play the role of a small parameter, and asymptotical analysis can be performed. Rigorous results were obtained in the field of (asymptotic) homogenization.

However, many scientific and industrial multiscale problems do not fall into the category of problems with scale separation. Intensive research on variety of numerical upscaling approaches was carried out in the last decade. At the same time, the Algebraic Multigrid Methods also evolved in this direction. The most active mathematical research in the field of numerical upscaling for flow problems is currently carried out in three directions: upscaling based on multigrid methods, upscaling based on multiscale finite element method and related approaches (MSFV, HMM, Variational Multiscale Method, etc.), and application of upscaling for solving multiscale industrial and environmental problems. Both, MG and multiscale FEM, provide a suitable framework for solving multiscale problems in the case of no scale separation. A number of papers devoted to multigrid and upscaling were published recently. A topic, related to both, AMG and multiscale FEM, is the so called Algebraic Multigrid-element method, AMGe. Methods like

Multiscale FEM, MsFEM; Multiscale Finite Volume, MSFV; Variational Multiscale Method; Heterogeneous Multiscale Method, are well understood now, at least when applied as numerical upscaling procedure for multiscale linear elliptic problems. Complementary to this, some of these methods became a popular choice as an ingredient of two level domain decomposition preconditioners for heterogeneous problems. The consecutive iterations fine-to-coarse and coarse-to-fine scale is a natural feature of the multigrid method. In the last decade it was shown that approaches like model reduction has a great potential for solving multiscale problems. Intensive research is carried out recently in the area of multiscale stochastic PDE. This include both, stochastic homogenization and numerical upscaling for SPDE. Researchers from most of these closely related directions were invited to participate in the discussions.

3. Topics

The discussion was restricted to continuous-to-continuous upscaling for elliptic problems with deterministic and stochastic coefficients. Scalar PDEs as well as systems of PDEs were considered. The field of numerical upscaling is still a very broad research field, and for the miniworkshop several well focused topics were defined.

- *Numerical upscaling approaches.* MsFEM, MSFV, VMS, Heterogeneous Multiscale Method, etc. versus AMG, AMGe, EMB, etc. Multilevel/Multigrid preconditioners for multiscale problems; Frameworks for solving multiscale problems: Similarities, differences and synergy.
- *Numerical upscaling approaches for stochastic multiscale problems*: what can be reused from deterministic numerical upscaling approaches? Reduced basis approaches for deterministic and stochastic multiscale problems. Multlevel approaches for stochastic problems.
- *Upscaling effective properties of heterogeneous media in the case of stochastic media.* Spectral methods versus Monte Carlo and Multilevel Monte Carlo: robustness, efficiency, etc.
- *Convergence results.* A priori and a posteriori estimates for deterministic problems. How to use a posteriori estimates for developing adaptive multilevel precondiotioners for multiscale problems. Convergence of the spectral methods and Monte Carlo type methods in the case of multiscale problems with stochastic coefficients.
- *Benchmarking and validation.* A careful specification of benchmark problems and validation approaches is needed in the field of numerical upscaling. A class of benchmark problems can be provided by asymptotic homogenization, when the latter is applicable. Defining benchmark tests for coupled multiscale problems with deterministic and stochastic coefficients will be discussed.

The thorough and concentrated consideration of the above topics by the qualified participants in the mini-workshop ensured a strong synergy effect. Interconnections between different approaches were identified, thus enriching each of them, and providing a background for new developments.

Mini-Workshop: Numerical Upscaling for Media with Deterministic and Stochastic Heterogeneity

Table of Contents