Mathematisches Forschungsinstitut Oberwolfach

Report No. 06/2013

DOI: 10.4171/OWR/2013/06

Moduli Spaces in Algebraic Geometry

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3 February – 9 February 2013

Abstract. The workshop on Moduli Spaces in Algebraic Geometry aimed to bring together researchers working in moduli theory, in order to discuss moduli spaces from different points of view, and to give an overview of methods used in their respective fields.

Mathematics Subject Classification (2010): 14D22.

Introduction by the Organisers

The workshop Moduli Spaces in Algebraic Geometry, organized by Dan Abramovich (Brown), Gavril Farkas (HU Berlin), Lucia Caporaso (Rome) and Stefan Kebekus (Freiburg) was held February 4–8, 2013 and was attended by 25 participants from around the world. The participants ranged from senior leaders in the field to young post-doctoral fellows and one advanced PhD student. The range of expertise covered areas ranging from classical algebraic geometry to mathematics inspired by string theory. Researchers reported on the substantial progress achieved within the last three years, discussed open problems, and exchanged methods and ideas. Most lectures were followed by lively discussions among participants, at times continuing well into the night. For a flavor of the range of subjects covered, a few of the talks are highlighted below.

Stable pairs and knot invariants. Rahul Pandharipande (ETH Zürich) reported on work of Shende, Oblomkov and Maulik concerning Hilbert schemes $\mathrm{Hilb}(C,n)$ of n points on a curve C with an isolated, planar singularity. Building on ideas of Pandharipande-Thomas and Diaconescu, Shende and Oblomkov proposed a relation between the Euler characteristics $\chi(\mathrm{Hilb}(C,n))$ and coefficients in the HOMFLY polynomial of the curve singularity link. This was recently established by Maulik.

Higher codimension loci in the moduli space of curves. Nicola Tarasca (Leibniz Universität Hannover) reported on results in his PhD Thesis on the calculation of the cohomology class of the codimension two Brill-Noether locus of curves with a pencil of degree k in the moduli space $\overline{\mathcal{M}}_{2k}$ of stable curves of genus $2k$. Remarkable here is that, while one has a large number of divisor class calculations on the moduli space, it is for the first time that a closed formula for a higher codimension locus on the moduli space is found.

Tautological rings of moduli space of curves. In a very impressive talk, Aaron Pixton (Princeton) proposed a rather amazing conjecture generalizing at the level of the moduli space $\overline{\mathcal{M}}_{g,n}$ the Faber-Zagier relations in the cohomology of the moduli space $\mathcal{M}_g$. The increase in complexity when passing from smooth to singular curves is considerable and it is a major step forward that a concrete prediction has been put forward. The field is facing an interesting change of paradigm, in the sense that the largely accepted Faber Conjectures predicting that the corresponding tautological rings of moduli of curves satisfy Poincare duality, are being replaced by new predictions, according to which the suitable generalizations of Faber-Zagier relations span all relations between tautological classed. It is already clear that in genus 24 the two conjectures rule out each other (whereas for $g < 24$ they are equivalent) and it will be interesting to monitor future developments.

Geometric compactifications of the moduli space of K3 surfaces. A classical unsolved problem of moduli theory asks for a modular compactification of the moduli space of polarized K3 surface. While several compactifications of the moduli space have been discussed, none of them is known to date to support a universal family. Bernd Siebert (Hamburg) reported on joint work Mark Gross (San Diego), Paul Hacking (Amherst) and Sean Keel (Austin) which might lead to a solution of this long-standing problem. Building on work of Gross-Siebert which uses Mirror symmetry to study degenerations of Calabi-Yau manifolds, there is hope to single out one particular toroidal compactification for which a family might exist. While many details still need to be filled in, and a discussion of the geometric and modular properties of the construction is still pending, this is a very exciting project which might eventually solve a classical problem.

The moduli stack of semistable curves. This is a development providing a glimpse of the lively discussions which happened at this very meeting. Jarod Alper (ANU) reported in the most timely manner possible on present joint work with Andrew Kresch (Zürich) on the structure of the moduli stack of semistable curves. One of the main questions one must ask about any stack is whether or not it is a global-quotient stack, or at least if it can be approximated by a global-quotient stack. A central example is the stack $\mathfrak{M}_g^{ss}$ of semistable curves, a keystone in constructing many moduli spaces. Kresch has shown that even the first stage of this stack, $\mathfrak{M}_g^{ss,\leq 1}$, is not a global-quotient stack, but Alper conjectured that $\mathfrak{M}_g^{ss}$ falls in a general class of stacks well-approximated by global-quotient stacks. Alper reported that this was established during this meeting by him and Kresch for $\mathfrak{M}_g^{ss,\leq 1}$, with strong evidence for the result to hold for the full moduli stack of semistable curves.

Moduli of slope-semistable bundles. Daniel Greb (Ruhr-Universität Bochum) reported on joint work with Matei Toma (Nancy), discussing wall-crossing and compactifications for moduli spaces of slope-semistable sheaves on higher-dimensional projective manifolds. Generalizing work of Joseph Le Potier and Jun Li, he constructed projective moduli spaces for slope-semistable sheaves by showing semiampleness of certain equivariant determinant line bundles. While the geometry of the resulting moduli spaces is presently only partially understood, these spaces are likely to shed new light on the question whether Tian's topological compactifications of moduli spaces of slope-semistable vector bundles admit complex or even algebraic structures.

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