Mathematisches Forschungsinstitut Oberwolfach

Report No. 60/2012

DOI: 10.4171/OWR/2012/60

Dynamics of Patterns

Organised by
Wolf-Jürgen Beyn, Bielefeld
Bernold Fiedler, Berlin
Björn Sandstede, Providence

16 December – 22 December 2012

Abstract. Patterns and nonlinear waves arise in many applications. Mathematical descriptions and analyses draw from a variety of fields such as partial differential equations of various types, differential and difference equations on networks and lattices, multi-particle systems, time-delayed systems, and numerical analysis. This workshop brought together researchers from these diverse areas to bridge existing gaps and to facilitate interaction.

Mathematics Subject Classification (2000): 35B40, 35P05, 65M12.

Introduction by the Organisers

Nonlinear waves and patterns arise in partial differential equations and in systems posed on infinite lattices. While nonlinear waves and patterns are important in many biological and physical applications, many of their properties are still poorly understood from a mathematical viewpoint. Some outstanding problems in this area are the long-time behavior and nonlinear stability of localized and nonlocalized patterns on unbounded domains, the development of numerical algorithms to compute patterns and to assess their stability, analytical and geometric approaches for waves in lattice dynamical systems, and their control-theoretic aspects. Techniques range from dynamical systems, the analysis of nonlinear partial and functional differential equations including multiscale problems, to spectral analysis and numerical methods for patterns. It is the pattern aspect which unifies these widely separate technical aspects and provides substantial links between the proposed key topics. Over the past few decades, dynamical-systems techniques and bifurcation theory have helped tremendously in extending our understanding of the formation and dynamics of nonlinear waves on spatially extended domains. Most of these efforts, however, are restricted to structures that respect an underlying periodic spatial lattice.

The goal of the workshop on "Dynamics of Patterns", which was organized by W.-J. Beyn (Bielefeld), B. Fiedler (FU Berlin), and B. Sandstede (Brown), was to bring together researchers from the aforementioned diverse areas to bridge existing gaps and to facilitate interaction. The formal scientific program consisted of 5 longer talks, 24 shorter talks, and 4 brief talks given by participating graduate students. No talks were scheduled in the afternoons on Tuesday, Thursday and Friday: instead, participants were encouraged to form small groups and discuss open problems in order to facilitate informal interaction. This worked very well, and several groups met to talk about a wide range of topics from new software tools to theoretical questions. Some specific topics were blow-up problems, lattice dynamical systems, front propagation, interaction of multi-dimensional patterns, delay equations, and functional-analytic tools in stability problems.

Despite much progress over the past decades, our understanding of when nonlinear waves exist and when they are stable under small perturbations is still limited. The talks by Ghazaryan, Hoffman, Hupkes, McCalla, Rademacher, and Veerman addressed a wide range of topics related to these questions. Some of these talks focused on studying models for combustion processes and chemical reactions that involve small parameters which affect the dynamical properties of fronts and pulses. Also discussed were existence and stability problems of planar fronts in lattice dynamical systems, which involved the development of new techniques suited to deal with the essential spectrum.

Since it is often difficult to prove the existence of waves rigorously in a given model, the development of efficient and accurate algorithms that can be used to find such structures numerically is important for applications. The talks by Hochbruck, Latushkin, Otten, and Rottmann-Matthes focused on recent developments in exponential integrators, which can be used for fast direct numerical simulations, and on techniques for analyzing methods that allow one to "freeze" a traveling wave to find its profile and determine its stability. Among the new results that were presented were approaches that deal with waves in hyperbolic PDEs.

Another area of considerable interest are generalized patterns and waves which arise when long-lived structures exhibit a complicated irregular spatial and/or temporal form. Among these structures are time-periodic breather solutions in wave equations, composite front solutions, generalized fronts that arise in systems with inhomogeneities or noise, turbulent patterns in pipe flow, and quasi-periodic as well as transient patterns in Faraday experiments. Recent progress in this area was presented in the talks by Barkley, Chirilus–Bruckner, Mallet–Paret, Matano, Nishiura, Pego, Rucklidge, and Tuckerman.

Multiscale problems provide a rich source of interesting patterns that are often difficult to analyze: examples are patterns that arise in ferromagnetic materials or in media with periodic imperfections that occur on small scales, and long-wave frequency and amplitude modulations of large-scale periodic structures. The talks by Matthies, Mielke, Otto, and G. Schneider addressed various challenges in this area, including the use of homogenization methods, the analysis of domain wall structures, and the derivation and validity of reduced modulation equations for wave structures.

Another topic that was of significant interest to participants is the dynamics and control of structures involving delay equations and networks. The talks by Atay, Luecken, I. Schneider, Schöll, Walther, and Wolfrum focused on synchronization, chimera solutions, and control of structures in coupled oscillator systems with delay, and on the dynamics of delay differential equations with state-dependent delays.

For several classes of systems, it is possible to describe not just certain nonlinear wave structures but, in fact, all long-lived solutions on the attractor. The talks by Beck, Ben–Gal, Gurevich, Krisztin, and Rocha outlined recent progress on the dynamics of metastable patters in the 2D Navier–Stokes equations, the dynamics of reaction-diffusion systems with hysteresis, and the surprising analogies of global attractors in one-dimensional reaction-diffusion equations and in delay differential equations.

Workshop: Dynamics of Patterns

Table of Contents