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Advanced Computational Engineering

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Abstract. The finite element method is the established simulation tool for the numerical solution of partial differential equations in many engineering problems with many mathematical developments such as mixed finite element methods (FEMs) and other nonstandard FEMs like least-squares, non-conforming, and discontinuous Galerkin (dG) FEMs. Various aspects on this plus related topics ranging from order-reduction methods to isogeometric analysis has been discussed amongst the pariticpants form mathematics and engineering for a large range of applications.

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Introduction by the Organisers

The finite element method is now an established simulation tool for the solution of partial differential equations that are related to the models in many engineering problems. It has an enormous range of applicability including elasticity, visco-elasto-plasticity, contact, fluid mechanics, heat conduction, acoustics, and electromagnetism. As the range of applications has grown it has become increasingly apparent that special features of each model can be exploited to improve the performance and robustness of the finite element method (FEM). This has led to many mathematically interesting developments including mixed FEMs and other non-standard FEMs like least-squares, non-conforming, and discontinuous Galerkin (dG) FEMs. Their use often aims at improved or robust discretizations that are superior to standard FEMs in many numerical simulations in solid, structural, and fluid mechanics and electromagnetics.

Discontinuous Galerkin (dG) schemes are more flexible than standard finite element formulations. Here the trial and test functions are piecewise polynomial functions that are not required to be continuous across element boundaries which leads to an increase in the number of unknowns. Nonetheless the extra flexibility can be advantageous for up-winding, for adaptivity, for overcoming locking, and for the design of simple schemes for fourth order problems. At present, the convergence analysis for dG FEMs has been carried out for simple model problems, but the corresponding analysis for more complicated problems was in the focus of the discussion amongst the participants. This was shown in recent results reported by Jay Gopalakrishnan who discussed a discrete inf-sup condition and showed superb applications to wave propagation in heterogeneous materials. Antonio Huerta made a comparative study of continuous and discountinuous Galerkin formulations that lead to some surprising results and thus proved the urge of further research in this area. Furthermore Eun-Jae Park gave an overview on the latest developments on hybrid dG and mixed FEM for problems with multiple scales.

While edge elements have proved very useful in electromagnetics applications, it was shown that they can also be applied to elasticity and flow problems using special weak forms. this was discussed in depth in the presentation by Joachim Schöberl who reviewed new classes of elements for elasticity and flow problems based on *curl* and *div div* spaces. This generated great interest, especially, since the related discretization schemes are accompanied by fast iterative solvers. Nevertheless significant questions remain about the best choice of elements and penalty parameters, a posteriori estimates and the robustness of the various schemes to non-conforming mesh adaptation.

Besides principle applications and the empirical investigation of the performance in practical real-life applications, the mathematical understanding of the non-standard FEMs was progressed during the meeting. Mira Schedensack presented comparison results which complemented earlier work by Dietrich Braess. For the simplest model problem, three different FEMs are equivalent in their performance up to multiplicative constants and up to data oscillations. Dietmar Gallistl discussed the optimality of adaptive mesh-refining strategies for the Stokes equations using a pseudo-stress formulation, while Hella Rabus pointed out the robustness in the optimality of non-conforming adaptive FEMs in elasticity; all three were Leibniz fellows. Finally, Thirupathi Gudi illustrated the medius analysis for fourth-order plate problems with mathematical arguments from the a priori and a posteriori error analysis.

In the whole it was impressive to note how much mixed finite element technologies have recently progressed by combining the deep knowledge of the properties of the mathematical spaces with technological aspects such as e.g. integration rules and element shapes.

The application of conforming and non-conforming mixed FEMs to finite strain problems with hyperelastic materials were discussed by Ferdinando Aurricchio. He illustrated many challenges related to the design of finite element spaces in combination with different incompressibility conditions. This has stressed pratical and mathematical challenges that constitute problems with non unique solutions.

Isogeometric analysis has gained interest over the last years since it opens up the possibility to go directly from CAD models to the analysis phase. Besides the classical NURBS based interpolations, new formulations with T-Splines and other mesh coupling techniques were explored in the talks by Robert L. Taylor and Yuri Bashilevs. Especially the application to fluid-structure interaction problems of wind turbines generated interest since this coupled modern discretizations techniques with the dynamics solution of large systems based on moving meshes. Besides being a promising new technology for science and engineering numerical simulation, the new method carries a rich mathematical structure still to be explored. Those presentations along with the one of Wolfgang Wall have documented the interest of using non-standard meshes in real industrial applications. It has been clear during the discussion that progresses in this hot topic of non-conforming meshes (precision, stability, optimality) can be made by merging the knowledge of the mathematical community with the one of the engineering science community.

Least-squares FEMs offer in general some advantages compared to other variational methods, such as, for instance, the inherent ellipticity of the governing functional or an a posteriori error estimator without additional costs. Even for equations with non-selfadjoint operators, the procedure leads to a symmetric and positive definite algebraic system, which can be solved efficiently with iterative solution strategies. Recent developments related to large strain models for hyperelastic materials were discussed for example in the talk by Alexander Schwarz. Those results can be generalized to even more complex physically nonlinear continuum mechanical problems; as e.g. quasi-incompressibility, elasto-viscoplasticity and anisotropic elasticity.

In a variety of complex applications, the discretization of the underlying boundary value problem could lead to several 100 million of unknowns. With todays demands to solve such problems in industrial and medical environments in real-time computing, one has to design reduced models for the design of process control algorithms and for the numerical simulation of processes with uncertainties. The scientific challenge is to simplify the model in such a way that certain modeling aspects are accurately described while non-relevant aspects are neglected. The question of modeling error and of output of interest are at the heart of reduction method and of progresses in uncertainty quantification. The later aspect has been particularly clear in the talk of Michael Ortiz. The certification of computation with guaranteed bounds in an uncertain context is one future topic of highest relevance in advanced engineering computation.

While there are established methods of order-reduction available in some fields (like e.g. proper orthogonal decomposition (POD) or singular value decomposition (SVD), reduced basis (RB) techniques, and Craig-Bampton methods in structural dynamics or model order reduction in differential equation systems), there exist only a very limited amount of methodologies that can be employed in order to reduce nonlinear or multi-scale problems. The lectures by Pierre Ladeveze addressed the proper generalized decomposition (PGD) method that has advantages in engineering application and that can be combined with error analysis for domain decomposition methods. Some of the modeling approaches include multi-scale models and uncertainty quantification procedures. Daniel Peterseim could provide a new analysis technique for up and down scaling effective for applications even without scale separation. All those different approaches were discussed vividly during the breaks and in the evenings. Other new methods emerge from the need to do real-time computations e.g. in computer assisted surgery, where Here Stefanie Reese and Adrien Leygue provided new insights. Especially the presentation by Adrien Leygue showed how to combine off-line with on-line computations in order to achieve real-time analysis tools. This was demonstrated by an impressive deformation analysis of a liver. Nevertheless these techniques are still in their beginnings but will open up new possibilities for many engineering and medical applications.

The interaction of mathematicians and engineers was extremely fruitful and provided definitions of new research directions. This workshop on Advanced Computational Engineering clearly demonstrated that the field is very active and currently enjoys great progress with many new important results. It became transparent during the workshop that new non-standard FEMs with higher-order approximations and a huge range of different partial differential equations have a large potential for future work. The improvement of sharp bounds for error control within adaptive methods, mathematical stability conditions – for example the nonlinear counterpart of the inf-sup condition – lead to valuable discussion amongst the participants from the different mathematical and engineering disciplines.

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