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Interplay of Analysis and Probability in Physics

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Abstract. The main purpose of this workshop was to foster interaction between researchers in the fields of analysis and probability with the aim of joining forces to understand difficult problems from physics rigorously. 52 researchers of all age groups and from many parts of Europe and overseas attended. The talks and discussions evolved around five topics on the interface between analysis and probability. The main goal of the workshop, the systematic encouragement of intense discussions between the two communities, was achieved to a high extent.

Mathematics Subject Classification (2000): 60xx, 70xx, 74xx, 82xx.

Introduction by the Organisers

Developments in the last few years have shown that hard problems from physics can be tackled by combining tools from analysis and probability. The main goal of this workshop was therefore to bring together researchers working on selected topics on the interface between analysis and probability and to encourage intensive discussions between them. The topics chosen evolve around a rigorous understanding of complex physical systems, some of which contain random inputs. The participants were mostly well-known researchers in either field with a profound interest in the other, but also a number of younger scientists at the beginning of their career attended. The particular mixture of people encouraged a high degree of interaction.

The workshop was a continuation of an Oberwolfach workshop held in December 2008. The general aim and spirit of the two workshops were similar, but the

concrete topics of the two workshops had only a limited overlap, and apart from the organisers just seven researchers attended both workshops.

The schedule of the workshop was as follows. Each of the five days was devoted to one of the topics (1) Optimal transport, (2) Discrete-to-continuum transitions in systems out of equilibrium, (3) Diffusion and mass transport in inhomogeneous media, (4) Surfaces and interface evolutions, (5) Phase transitions in many-body systems. In the morning, a 60-minutes talk with survey character was delivered by a senior scientist, followed by two to five research talks of 40 minutes each. One further session was devoted to nine 5-minute talks by young scientists. In this way, almost all younger participants had the opportunity to present their work. Every talk was followed by a discussion and a number of questions from the audience.

The main goal of the workshop was remarkably well achieved. Evidence for this were intense cross-community discussions and an unusually high number of questions and remarks after every talk coming equally from representatives of both fields. The interaction between researchers in analysis and probability was much more intense than in the 2008 workshop. This may be due to the choice of the topics and of the participants, but we believe it mainly reflects the success of this workshop series (and other recent events) in closing the gap between the two research communities.

We now turn to a short description of the main contributions of the workshop. The first day was opened with a survey by Felix Otto on the interaction between gradient flows and hydrodynamic limits. Then Giuseppe Savaré gave an overview of the recent developments in metric-measure theory, and its connection to heat flows and entropy gradient flows. Yann Brenier presented a remarkable derivation of a one-dimensional compressible Navier-Stokes equation from a system of random walkers that is replaced in ordered sequence at every time step. Max von Renesse discussed stochastic analysis on the Wasserstein space and the Wasserstein diffusion. Marco di Francesco presented an extension of the Wasserstein gradient flow theory to a class of partial differential equation with singular potential. Finally Wilfrid Gangbo described recent developments in the theory of Vlasov systems.

On the second day Frank den Hollander presented recent results on the behaviour of a random copolymer near an interface between two immiscible solvents, showing variational characterizations of the phase diagram. Jean-Dominique Deuschel demonstrated a method to construct Markov chains approximating a given diffusion process with non-symmetric diffusion coefficients. Gero Friesecke presented a novel application of optimal transport theory in the context of density functional theory. Lorenzo Bertini explained how to extend the Donsker-Varadhan large deviation principle for the empirical measure by considering also the empirical flow of a Markov process. Pierre Mathieu described a remarkable scaling limit for a random walk in a trap environment, and Greg Pavliotis presented a deterministic coupled system consisting of a classical particle in contact with a heat bath, where the heat bath is modelled as a wave equation.

The third day started with Errico Presutti highlighting some aspects of two types of phase transitions for classical many-body systems with potentials of

Lennard-Jones type: solid-gas and fluid-gas. He put some emphasis on low-temperature aspects and connected up with optimality results derived at zero temperature. An approximate description of the first type of transitions at coupled low temperature and low density, using a large-deviation ansatz, was presented by Sabine Jansen. Daniel Ueltschi considered probability measures on the set of partitions, which are motivated by the study of the interacting Bose gas, but have many more interesting applications in statistical physics.

The fourth day was opened by Claudio Landim, who presented and compared the hydrodynamics of three interesting classes of random particle dynamics with exclusion and explained the differences in the structure of the dynamics and how they are reflected in the nature of the limiting differential equation. Then Cristian Giardinà described the concept of duality and an example application, in which the duality permits the exact solution of a class of interacting diffusions. Tony Lelièvre explained two methods to use coarse-graining in metastable systems for the efficient calculation of averages. This was followed by our session for young researchers, before Dirk Blömker addressed the question of stabilization of stochastic partial differential equations by additive noise, with applications to the Swift-Hohenberg equation and a Burgers' equation.

On the final day Andrey Piatnitski explained homogenization for certain types of parabolic operators with subdiffusive, diffusive and superdiffuse rescaling of the time variable. Aaron Yip presented a crystal growth model, in particular the peculiarities at the interface of the arising optimal shape, the Wulff shape. Antal Jarai gave a survey of the Markovian dynamics of sandpile models and analysed minimal recurrent configurations on unboundedly growing graphs. Omar Lakkis described an approach to error estimates in stochastic partial differential equations in L^∞- and L^p-norms, with applications to the Allen-Cahn equation with additive noise. Finally, Karsten Matthies used a collision tree approach to derive a simplified Boltzmann equation from a system of deterministic particles with random positions and annihilation dynamics.

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