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Explicit Versus Tacit Knowledge in Mathematics

Organised by
Tom Archibald, Burnaby
Jeanne Peiffer, Paris
Norbert Schappacher, Strasbourg

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Abstract. This workshop aimed to bring together an international group of historians of mathematics to reflect upon the role played by tacit knowledge in doing mathematics at various times and places. The existence of tacit knowledge in contemporary mathematics is familiar to anyone who has ever been given an idea of how a particular proof or theory "works" by a verbal analogy or diagrammatic explanation that one would never consider publishing. Something of it is felt by every student of mathematics, when the process of learning mathematics often amounts to training the right reflexes. In more advanced contexts, the tacit understanding that a particular technique, instrument or approach is "the one to use" in a given circumstance gives another familiar instance. Tacit knowledge, a term introduced by the philosopher M. Polanyi, contrasts with the explicit knowledge that in almost all historical mathematical cultures is associated with mathematical text. The workshop invited a use of the categories of tacit and explicit knowledge to achieve a better knowledge of how mathematical creation proceeds, and also of how cultural habits play a tacit role in mathematical production. The meeting intended to offer the possibility of significant innovation and enrichment of historical method, as well as new and compelling insight into the process of creating mathematics in different times and places. The meeting was intended to afford the opportunity for a presentation of selected case studies by leading experts and new scholars. In retrospect, as we hope these abstracts show, the results promise to be of significant interest not only to historians, but to the mathematical community more broadly.

Mathematics Subject Classification (2000): 01A.

Introduction by the Organisers

The aim of this workshop was to bring together an international group of historians of mathematics to reflect upon the role played by *tacit, as opposed to explicit knowledge* in doing mathematics at various times and places. Methodological discussions on the use of this concept alternated with specific case studies from the history of mathematics. The aim was to allow a better understanding of mathematical practices in given contexts. The theme impinges on the transmission of existing mathematics as well on the creation of new theories and results.

The existence of tacit knowledge in contemporary mathematics is familiar to anyone who has ever been given an idea of how a particular proof or theory "works" by a verbal analogy or diagrammatic explanation that one would never consider publishing. Something of it is felt by every student of mathematics, when the process of learning mathematics often amounts to training the right reflexes. In more advanced contexts, the tacit understanding that a particular book or paper or approach is "the one to use" in a given circumstance gives another familiar instance. The theme was specifically chosen for this meeting on the history of mathematics in view of its inspirational and unifying potential, and in the ways that it promised to shed light on methods for understanding mathematical texts and practices of the past. Originally, our plan was to look at cases that range from the most ancient history of mathematics to current developments. We include here the original list of examples, and the reader can compare this to the actual papers, which achieved a comparable breadth while highlighting rather different features:

- The difference between algorithmic mathematics (like in ancient Mesopotamia or medieval China) and proof-oriented mathematics in the Euclidean tradition and the intermediate stages, like Chinese two-column algorithmic texts which are proof-driven but not in the Euclidean style are all too often analyzed without taking into account the parts of the practice that remain tacit and are not spelled out in the text, contributing thus to give a biased image of that difference.
- Tacit knowledge is present in various ways throughout the mathematical exchanges of the seventeenth century. Correspondence by letters included knowledge on how to write a letter, without spelling out the rules of letter writing. In cases where these tacit codes were not applied, it is interesting to give an interpretation of this step aside. More generally, tacit rules of scientific exchange dictated what was to be made explicit or public in a mathematical proof, and which parts were not. On the mathematical level, curves were identified by a catalogue of properties, which was never explicitly listed in its entirety. For instance, as soon as a curve was found to have the property that its subtangent is the double of the abscissa, it was identified with a parabola.
- A good deal of the development of mathematics in the nineteenth and twentieth centuries can be viewed as a process of making the practice of mathematics increasingly explicit, thereby reducing the amount of tacit

knowledge and thus opening up a wide space of rational discussion and achievements. However, this tendency to greater technical explicitness, which is evident in the typical manuscripts posted by mathematicians on ArXiv every day, may induce historians of mathematics to neglect the persistence of tacit knowledge in the most recent mathematics. The identification of such tacit elements seems capable of affording significant insights into the development of mathematics today.

- Similarly, several large scale mathematical enterprises of the last 100 years like Bourbaki's *Éléments de mathématique* or – in a different manner – computer-based mathematical research, like the more recent projects towards automated theorem proving (ATP), appear at first as signposts of a massive pushing back of tacit knowledge. Looking more closely, however, at details like the occasional warning signs in the margins of Bourbaki's volumes, or at problems related with the user interface, one sees that these undertakings carry in fact their own heavy collection of tacit mathematical practice.
- Developments in the history of mathematics are often loosely described as moving from approximate, incompletely understood treatments, to fully explicit, formal statements and their rigorous proofs. (See for instance Breger's contribution to [1].) Paying attention to the kind of tacit knowledge which is mobilized before and after such a development often provides a much more satisfactory analysis of the historical process than the mere confrontation of precise *versus* imprecise methods. A case in point is the rewriting of Algebraic Geometry in the first half of the twentieth century. In a 1926 letter to Hermann Weyl, Salomon Lefschetz significantly described the Italian school of Algebraic Geometry, not as lacking rigor, but as requiring "a terrible entraînement ". Later attempts, by Francesco Severi and others, to defend their classical Algebraic Geometry against growing criticism would invariably insist on the fact that all those technical assumptions or arguments which the modern algebraists could not find in the Italian papers where indeed tacitly assumed, and well-known to all geometers raised in the Italian school. The question whether the category of tacit knowledge may render such arguments historically convincing appears quite difficult, and can only be decided by very detailed case studies.
- In contemporary mathematics, blogs and Wikis – the most famous probably being Terence Tao's – currently provide an extended form of oral culture in which less formal, formerly tacit approaches are written down and opened to a broad mathematical public according to shifting and variable rules.

The term "tacit knowing" or "tacit knowledge" which we explored here in its bearing on the history of mathematics, comes from a philosophical context, but has been mobilized before for the history of science. Michael Polanyi introduced "tacit knowing", or "tacit knowledge" in order to describe abilities which cannot be fully described or explained (see [4]). In the history of science, the concept has been mobilized in the study of the craft aspects of experimental science from the seventeenth century to the present day. The philosophical theory of tacit knowledge has been much discussed over the years for instance also in the context of mathematical education and curricula, which is not the purpose of the workshop proposed here. More recently, the sociologist Harry Collins reassessed this notion in [2], in particular distinguishing several types of tacit knowledge.

The theory of tacit knowledge marks a counterpoint to the "ideal of wholly explicit knowledge" which took shape through the scientific revolution of the seventeenth century. Among the different interpretations which have been given of the concept of "tacit knowledge", from a conscious under-articulation or deliberate secrecy to the strong thesis that there are specific kinds of knowledge that cannot in principle be fully articulated – the standard example being here riding a bike – the application to the history of mathematics will focus on the weak sense: tacit knowledge is what mathematicians selectively conceal, avoid articulating or under-articulate, consciously or not. This does include the possible concealment of information by mathematicians competing with others, as well as the case of descriptions which are left incomplete because their authors assume, or know by experience, that their readers share a certain knowledge with them. Tacit knowledge is then built on experience or action, and cannot be fully described by rules or words. It concerns any type of knowledge or skill used as subsidiary to the performance and control of a mathematical task. The notion of tacit knowledge could be applied to the history of mathematics, as suggested by Breger ten years ago who used the greater level of abstraction created by the ongoing development of mathematics to detect tacit elements in earlier texts. This is a challenging thesis but obviously history of mathematics should not be reduced to just re-reading old texts through the spectacles of more modern mathematical achievements.

At this point, more recent methods in the history of mathematics come to the rescue: following a tradition that can be traced back to Ludwig Wittgenstein and other authors of the 1930s and 1940s, the second half of the twentieth century has seen authors such as Imre Lakatos, Paul Feyerabend, and Hans-Jörg Rheinberger placing the detailed analysis of scientific practice at the heart of history of science. This goes hand in hand with the realization that tacit scientific knowing is acquired by the individual scientist through a social context or network whose members share a common know-how. Although unstated know-how tends to be difficult to identify in a single mathematical text, shared tacit knowledge or know-how is more accessible, often by way of comparison with other local mathematical cultures or broader networks. It also tells a lot about mathematical (and strategic) practices in a specified time period.

In the case of mathematics, Epple has adapted Rheinberger's approach to the history of mathematics in his book [3] on the history of knot theory. The notions of *epistemic objects* and *epistemic techniques* are his key concepts to describe the ways of the active researchers to handle the complex web of established theories ready for use, formal and informal operational skills to deal with new phenomena, and often vague general ideas about the kind of mathematical object under focus.

Furthermore, the mathematical tools made use of in specific contexts or sites are in most cases abstract techniques or objects, but may also be material devices, from the measuring rod and compass to the analog integrator and computer. In the Renaissance and early modern periods, the design and use of such instruments was a core feature of mathematical practice, and the tacit knowledge involved in acquiring the techniques of use or design was considerable. Yet such knowledge has left historical evidence: Albrecht Dürer, most famously, tried to describe explicitly what perpective artists were actually doing, including the gestures transmitted through long workshop traditions. One aim of the conference will be to assess the degree of continuity between these older traditions and those in evidence in more recent mathematical practice.

Our main objective for the conference proposed here was thus to use the peculiar bias of the distinction between tacit and explicit knowledge in order to re-invigorate discussions about how the analysis of social networks on the one hand and of the research practice of mathematicians on the other come together to afford a close-up understanding of the historical process which we call mathematics. Last but not least we hoped it would allow a better understanding of how mathematical practices depend on larger cultural habits, or are embedded in larger cultural contexts, including language, writing cultures, literary and rhetorical devices, and craft knowledge.

The abstracts below show that the chosen theme has proved inspiring for most of the speakers, whom it enabled to highlight aspects of the production and transmission of mathematics which have often been neglected. The use of instruments, which may imply a lot of bodily skills which can scarcely be transmitted through words, is typically an example of such an understudied aspect of mathematics. The practice of skills was also one of the starting points of Michael Polanyi in his 1958 *Personal Knowledge*, as Jeanne Peiffer recalled in her short introduction to the workshop. Before describing the tacit component of The Art of Knowing, Polanyi suggests to grasp "the nature of the scientist's personal participation by examining the structure of skills" ([4, 49]and as his clue for this investigation, he takes what he calls the well-known fact "that the aim of a skilful performance is achieved by the observance of a set of rules which are not known as such to the person following them" (ibid.). For Polanyi, an art, a skill, which cannot be specified in detail – think at the famous example of riding a bike – cannot be transmitted by prescription, since no such prescription exists. It can be passed on only by example from master to apprentice. It follows that an art which has fallen into disuse for the period of a generation is altogether lost. And here questions for the historian come in, mostly methodological questions. How can we, as historians, recover not specified, not explicated skills, arts or knowledge ? Besides methodological reflections, a whole range of case studies have been presented by the participants of the workshop which have shown the various forms of tacitness.

Norbert Schappacher in his introduction briefly reminded the audience of Michael Polanyi's record:
(a) As a researcher, in particular as director of the chemical-kinetics research group in Fritz Haber's *Kaiser-Wilhelm-Institute* for physical chemistry and electrochemistry in Berlin-Dahlem starting in 1923; see [5], esp. chap. 2 and 3; cf. Polanyi's ranking among the leading scientists of the Kaiser-Wilhelm-Gesellschaft at the time in [6], vol. 2, p. 1254.
(b) As a thinker on economic theory, fighting the rather marxist tendencies of his brother Karl. Hachtmann in [5], vol. 1, pp. 31-32, points to text of Polanyi's from as early as 1930, on the return of investment into the sciences (*Rentabilität der Wissenschaften*) which kind of anticipated, in the concrete context of the *Kaiser-Wilhelm-Institutes* [KWIs] menaced by spending cuts after the big economic crisis, Pierre Bourdieu's later theory of the exchangeability of *actual, cultural, symbolic*, and *social* capital.
(c) Of the later unfolding of Polanyi's ideas of personal later tacit knowledge, driven by a desire (probably partly inspired by Ludwik Fleck) to balance Popper's so-called *Logic of scientific discovery* by more genuine descriptions of scientific practice, and by a more Gestalt-theoretic approach of scientific work, and its part in human culture at large.

More to the point of the subject of this meeting, i.e., the history of mathematics, Polanyi's letter to Lakatos of August 14, 1961 (from the Archives of the London School of Economics; thanks to H.J. Dahms for sharing it with us) was quoted, written in response to reading a draft of Lakatos' *Proofs and Refutations.*There one reads in particular : "If you are interested to find out as I am, how it can be that these procedures of acquiring what we call knowledge, do in fact lead to something that is knowledge, though it is, and must remain, impossible to define these procedures, or set up criteria of their success, without appealing to powers which are defined by no rules, then one feels that to speak of conjectures and refutations etc. as answering my question, is to beg it."

References

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[4] Michael POLANYI, *Personal knowledge. Towards a post-critical philosophy.* London 1969.
[5] Mary-Jo NYE, *Michael Polanyi and his Generation. Origins of the social construction of science.* Chicago & London 2011.
[6] Rüdiger Hachtmann, *Wissenschaftsmanagement im "Dritten Reich". Geschichte der Generalverwaltung der Kaiser-Wilhelm-Gesellschaft.* 2 volumes. Göttingen 2007.

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